Combinatorics

No repetition

1) Permutations of n elements**: P(n)= n != n(n-1)(n-2)…3 $\circ$ 2 $\circ$ 1** by definition is: **0!=1**

2) Variations **V_k^n =n(n-1)(n-2)…(n-k+1)**

***n* is the number of objects from which you can choose and *k* is the number to be chosen,**

3) Combination $\mathbf{C}_k^n=\binom{n}{k}=\frac{V_k^n}{k!}=\frac{n(n-1)...(n-k+1)}{k!}$

still applies: $\binom{n}{0}=\binom{n}{n}=\mathbf{1}$ **,** $\binom{n}{1}=\binom{n}{n-1}=\mathbf{n}$ **,** $\binom{n}{k}=\binom{n}{n-k}$

To be repeated

1) Number of permutations of n elements of which k is equal : $\mathbf{P_k(n)}=\frac{n!}{k!}$
2) Variations $\overline{V_k^n}$ $=\mathbf{n^k}$
3) Combination $\overline{C_k^n}=\binom{n+k-1}{k}$

First law of countdown: If one event can be implemented on **m** ways, and otherwise **n** ways, then one of them can be implemented on **m + n** ways

Second law of countdown: If one event can be implemented on **m** ways, and a second event on **n** ways, then both events can at the same time implemented in ways **mn** .

How to recognize whether the P, V or C?

Shall be provided with the set of n different elements. **If you work with all n elements,** or make all possible different layouts of the n elements, then we'll use **permutations.**

If you need to create all the subsets of k different elements where **the order of elements** is **important,** then we will use **variations.**

If you need to create all the subsets of k different elements where **the order of elements** is **not important,** then we will use **combinations.**

EXAMPLES:

1) We have two basic characters: **.** and **-**
If one sign consists of a maximum five basic characters, how much characters can we make?

Solution:

- We have two characters, . and - (point and dash) $\longrightarrow$ $n = 2$
- We have 5 situation:

1) If you have only 1 sign $\rightarrow \bar{V}_1^2$
2) If you have 2 characters $\rightarrow \bar{V}_2^2$
3) If you have 3 characters $\rightarrow \bar{V}_3^2$
4) If you have 4 characters $\rightarrow \bar{V}_4^2$
5) If you have 5 characters $\rightarrow \bar{V}_5^2$

And the final solution:

$$\bar{V}_1^2 + \bar{V}_2^2 + \bar{V}_3^2 + \bar{V}_4^2 + \bar{V}_5^2 =$$
$$2^1 + 2^2 + 2^3 + 2^4 + 2^5 =$$
$$2 + 4 + 8 + 16 + 32 = 62$$

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2) Determine the number of natural numbers less then 10 000, which can be formed from the digits 0,1,2,3,4,5.

Solution:

Think:

Requested numbers can be:

1) one- digit
2) two- digit
3) three-digit
4) four - digit
5) 5- digit

We have 6 numbers: 0, 1, 2, 3, 4, 5 and numbers can be repeated. $\longrightarrow$ $\overline{V}_k^n$ **=n^k^** $= n^k$

Must be careful : **0 can not be in the first place!**

1) one- digit $\overline{V}_{1}^{5}=5$

2) two- digit $\rightarrow \overline{V}_{2}^{6}-\overline{V}_{1}^{6}=6^{2}-6^{1}=30$

3) three-digit $\rightarrow \overline{V}_{3}^{6}-\overline{V}_{2}^{6}=6^{3}-6^{2}=180$

4) four – digit $\rightarrow \overline{V}_{4}^{6}-\overline{V}_{3}^{6}=6^{4}-6^{3}=1080$

5) 5- digit $\rightarrow \overline{V}_{5}^{6}-\overline{V}_{4}^{6}=6^{5}-6^{4}=6480$

So the final solution is: $5+30+180+1080+6480=7775$

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3) On how many different ways can distribute 5 boys and 5 girls in the picture line of 10 chairs so that the two boys never sit next to one another?

Solution:

Think:

As has 10 seats and 2 boys may not be one to the other, it means that the schedule is one boy one girl.

boy girl boy girl boy girl boy girl boy girl

- Possibility for boys is 5!
- Possibility for girls is 5!

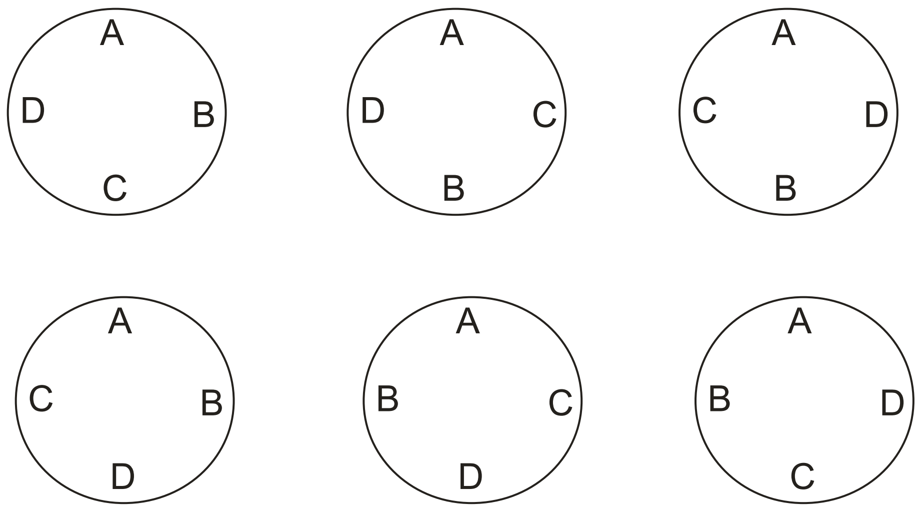
But we have to think, that in the first place can be girls!

girl boy girl boy girl boy girl boy girl boy

And the number of options: $2\cdot 5!5!=2\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1=2\cdot 120\cdot 120=28800$

4) Four people should be place on the circle. How we can do that?

Solution:



So, there are 6 possibilities!

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5) Find all four – digit numbers that begin with 2 and end with 7?

Solution:

These are the numbers 2 ▒▒ 7, where instead of boxes can be numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

So, solution is: $\overline{V}_1^2 = 10^2 = 10 \cdot 10 = 100$

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6) How many numbers between 3000 and 6000, which ends with 3 or 7?

Solution:

→ numbers starting with 3 are:

3 ▒▒ 3 → $\overline{V}_2^{10} = 10^2 = 100$

3 ▒▒ 7 → $\overline{V}_2^{10} = 10^2 = 100$

→ numbers that begin with 4:

4 ▒▒ 3 → $\overline{V}_2^{10} = 10^2 = 100$

4 ▒▒ 7 → $\overline{V}_2^{10} = 10^2 = 100$

Similarly, 5 ▒ ▒ 3 → 100 numbers
5 ▒ ▒ 7 → 100 numbers

Finally solution: 100·6=600 numbers

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7) How has three-digit numbers that are divisible by 5 ?

Solution:

We have 900 three-digit numbers from 100 to 999.

Since every fifth is divisible with 5 ,ranging from 100 ,there are 900:5 = 180 numbers!

8) Basketball team consists of 5 guards, 4 centers and 3 forwards. On how many ways can be quintet put together if it must be at least 2 guards and at least one center?

Solution:

Since the task say that in five to play at least must be 2 guards and at least one center, that give us more possibilities:

1) 2 guard, 1 center, 2 forward $\rightarrow C_2^5 \cdot C_1^4 \cdot C_2^3$
2) 2 guard, 2 center, 1 forward $\rightarrow C_2^5 \cdot C_2^4 \cdot C_1^3$
3) 2 guard, 3 center $\rightarrow C_2^5 \cdot C_3^4$
4) 3 guard, 1 center, 1 forward $\rightarrow C_3^5 \cdot C_1^4 \cdot C_1^3$
5) 3 guard, 2 center $\rightarrow C_4^5 \cdot C_2^4$
6) 4 guard, 1 center $\rightarrow C_4^5 \cdot C_1^4$

Now the number of all options:

$$C_2^5 \cdot C_1^4 \cdot C_2^3 + C_2^5 \cdot C_2^4 \cdot C_1^3 + C_2^5 \cdot C_3^4 + \cdot C_3^5 \cdot C_1^4 \cdot C_1^3 + \cdot C_3^5 \cdot C_2^4 + C_4^5 \cdot C_1^4 =$$

$$\binom{5}{2}\binom{4}{1}\binom{3}{2}+\binom{5}{2}\binom{4}{2}\binom{3}{1}+\binom{5}{2}\binom{4}{3}+\binom{5}{3}\binom{4}{1}\binom{3}{1}+\binom{5}{3}\binom{4}{2}+\binom{5}{4}\binom{4}{1}=540 \text{ possibilities}$$