**1. Examine function and draw a graph :** $y=\sqrt[3]{x^2-x^3}$

***Domain***

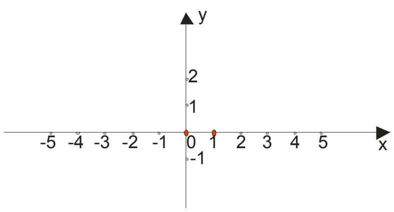
This function is defined everywhere because there are no fractions and the third root is everywhere defined

So: $D_f=(-\infty,\infty)$

This tells us that there is no vertical asymptote.

***Zero function***

$$y=0 \to x^2-x^3=0 \to x^2(1-x)=0$$
$$x=0; x=1$$




***Sign function***

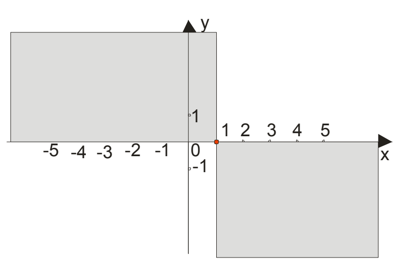
As always, think first of which depends on sign function ?

$$y=\sqrt[3]{x^2-x^3}=\sqrt[3]{x^2(1-x)}$$

Only 1-x , then is :

| | -∞ | 1 ∞ |
|---|---|---|
| 1-x | + | — |
| y | + | — |

the diagram is




***Parity***

$$f(-x)=\sqrt[3]{(-x)^2-(-x)^3}=\sqrt[3]{x^2+x^3}$$

***Extreme values (max and min) and monotonic function ( increasing and decreasing )***

$y=\sqrt[3]{x^2-x^3}$ easier for us to ask if function look like :

$$y=(x^2-x^3)^{\frac{1}{3}}$$

$$y`=\frac{1}{3}(x^2-x^3)^{\frac{1}{3}-1}\cdot(x^2-x^3)`$$

$$y`=\frac{1}{3}(x^2-x^3)^{-\frac{2}{3}}\cdot(2x-3x^2)$$

$$y`=\frac{1}{3}\frac{2x-3x^2}{\sqrt[3]{(x^2-x^3)^2}}$$

$$y`=0\rightarrow 2x-3x^2=0\rightarrow x(2-3x)=0\rightarrow x=0\vee x=\frac{2}{3}$$

<u>For x = 0</u>

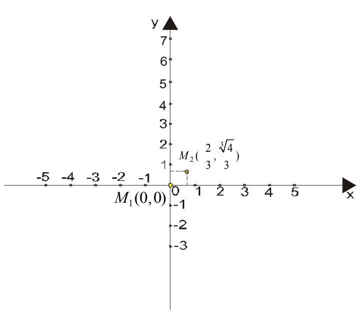
$$y=\sqrt[3]{0-0}=0$$

<u>For x = $\frac{2}{3}$</u>

$$y=\sqrt[3]{(\frac{2}{3})^2-(\frac{2}{3})^3}=\sqrt[3]{\frac{4}{9}-\frac{8}{27}}=\sqrt[3]{\frac{4}{27}}=\frac{\sqrt[3]{4}}{3}$$

*So*:

$$M_1(0,0);M_2(\frac{2}{3},\frac{\sqrt[3]{4}}{3})$$




| | -∞ | 0 | $\frac{2}{3}$ ∞ |
|---|---|---|---|
| x | − | + | + |
| 2-3x | + | + | − |
| y` | − | + | − |
| | ↘ | ↗ | ↘ |

***convexity and concavity***

$$y=\sqrt[3]{x^2-x^3}$$

$$y`=\frac{1}{3}(x^2-x^3)^{-\frac{2}{3}}\cdot(2x-3x^2)$$

$$y``=\frac{1}{3}[((x^2-x^3)^{-\frac{2}{3}})`\cdot(2x-3x^2)+\ (x^2-x^3)^{-\frac{2}{3}}\cdot(2x-3x^2)`]$$

$$y``=\frac{1}{3}[-\frac{2}{3}(x^2-x^3)^{-\frac{5}{3}}\cdot(2x-3x^2)\cdot(2x-3x^2)+\ (x^2-x^3)^{-\frac{2}{3}}\cdot(2-6x)]$$

After careful calculation :

$$\boxed{y``=-\frac{2}{9(1-x)^{\frac{5}{3}}\cdot x^{\frac{4}{3}}}}$$

**For x > 1 is y`` > 0 and for x < 1 is y`` < 0**

***Asymptote function (behavior functions at the ends of the field definition)***

As we have already concluded, the function has no vertical asymptote!

<u>Horizontal asymptote</u>

$$\lim_{x\to+\infty}\sqrt[3]{x^2-x^3}=\lim_{x\to+\infty}\sqrt[3]{x^3(\frac{1}{x}-1)}=\lim_{x\to+\infty}x\cdot\lim_{x\to+\infty}\sqrt[3]{(\frac{1}{x}-1)}=\infty\cdot(-1)=-\infty$$

$$\lim_{x\to-\infty}\sqrt[3]{x^2-x^3}=\lim_{x\to-\infty}\sqrt[3]{x^3(\frac{1}{x}-1)}=\lim_{x\to-\infty}x\cdot\lim_{x\to-\infty}\sqrt[3]{(\frac{1}{x}-1)}=-\infty\cdot(-1)=+\infty$$

<u>Oblique asymptote</u>

$$k=\lim_{x\to\infty}\frac{\sqrt[3]{x^2-x^3}}{x}=\lim_{x\to\infty}\frac{\sqrt[3]{x^3(\frac{1}{x}-1)}}{x}=\lim_{x\to\infty}\frac{\cancel{x}\cdot\sqrt[3]{(\frac{1}{x}-1)}}{\cancel{x}}=\lim_{x\to\infty}\sqrt[3]{(\frac{1}{x}-1)}=0-1=-1$$

$$n = \lim_{x\to\infty}[f(x) - kx]$$

$$n = \lim_{x\to\infty}[\sqrt[3]{x^2 - x^3} + x]$$ here we must use : $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

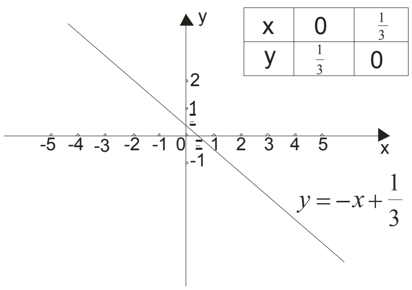
$$n = \lim_{x\to\infty}(\sqrt[3]{x^2 - x^3} + x)\cdot\frac{(\sqrt[3]{x^2 - x^3})^2 - \sqrt[3]{x^2 - x^3}\cdot x + x^2}{(\sqrt[3]{x^2 - x^3})^2 - \sqrt[3]{x^2 - x^3}\cdot x + x^2} = \lim_{x\to\infty}\frac{(\sqrt[3]{x^2 - x^3})^3 + x^3}{(\sqrt[3]{x^2 - x^3})^2 - \sqrt[3]{x^2 - x^3}\cdot x + x^2}$$

$$n = \lim_{x\to\infty}\frac{x^2 - x^3 + x^3}{\sqrt[3]{x^4 - 2x^5 + x^6} - \sqrt[3]{x^3(1 - \frac{1}{x})}\cdot x + x^2} = \lim_{x\to\infty}\frac{x^2}{\sqrt[3]{x^6(\frac{1}{x^2} - \frac{2}{x} + 1)} - x\cdot\sqrt[3]{(1 - \frac{1}{x})}\cdot x + x^2}$$

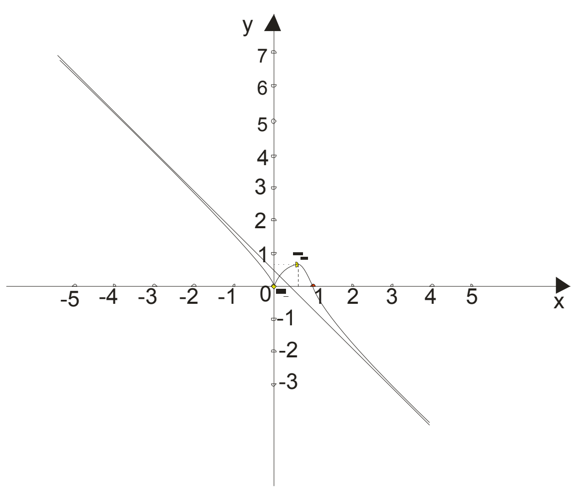
$$n = \lim_{x\to\infty}\frac{\cancel{x^2}}{\cancel{x^2}[\sqrt[3]{(\frac{1}{x^2} - \frac{2}{x} + 1)} - \sqrt[3]{(1 - \frac{1}{x})} + 1]} = \frac{1}{1 - (-1) + 1} = \frac{1}{3}$$

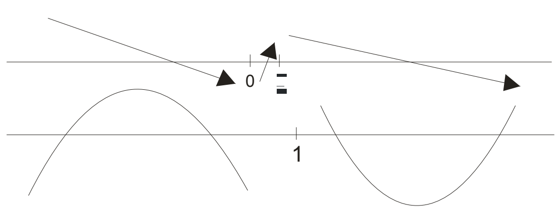
$$\boxed{y = -x + \frac{1}{3}}$$

| x | 0 | $\frac{1}{3}$ |
|---|---|---|
| y | $\frac{1}{3}$ | 0 |




And finally:



**2. Examine function and draw a graph :** $y = \dfrac{x-2}{\sqrt{x^2+2}}$

***Domain***

Here we observe two conditions:

$\sqrt{x^2+2} \neq 0$ and $x^2+2 \geq 0$

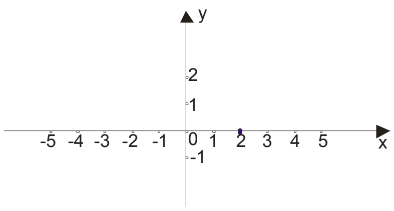
So: $x^2+2 > 0$

This is obviously true for any real x, so: $D_f = (-\infty, \infty)$

And here we conclude that the function has no vertical asymptote.

***Zero function***

y=0 for x-2 = 0 , then we have: x = 2

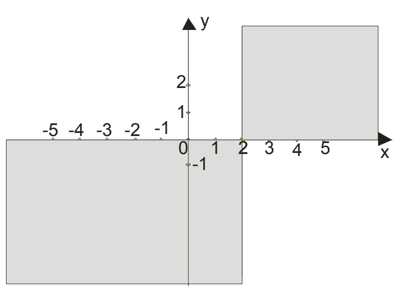



***Sign function***

$$y = \frac{x-2}{\sqrt{x^2+2}}$$

Kako je izraz u imeniocu uvek pozitivan, zaključujemo da znak zavisi samo od brojioca...

| | $-\infty$ … 2 | 2 … $\infty$ |
|---|---|---|
| x-2 | – | + |
| y | – | + |




***Parity***

$$f(-x) = \frac{-x-2}{\sqrt{(-x)^2+2}} = \frac{-x-2}{\sqrt{x^2+2}} \neq f(x)$$

***Extreme values (max and min) and monotonic function ( increasing and decreasing )***

$$y=\frac{x-2}{\sqrt{x^2+2}}$$

$$y`=\frac{(x-2)`\cdot\sqrt{x^2+2}-(\sqrt{x^2+2})`\cdot(x-2)}{(\sqrt{x^2+2})^2}$$

$$y`=\frac{1\cdot\sqrt{x^2+2}-\frac{1}{2\sqrt{x^2+2}}\cdot(x^2+2)`\cdot(x-2)}{x^2+2}$$

$$y`=\frac{1\cdot\sqrt{x^2+2}-\frac{1}{\not 2\sqrt{x^2+2}}\cdot\not 2x\cdot(x-2)}{x^2+2}$$

$$y`=\frac{\frac{(\sqrt{x^2+2})^2-x(x-2)}{\sqrt{x^2+2}}}{x^2+2}$$

$$y`=\frac{x^2+2-x^2+2x}{(x^2+2)\sqrt{x^2+2}}$$

$$y`=\frac{2+2x}{(x^2+2)\sqrt{x^2+2}}$$

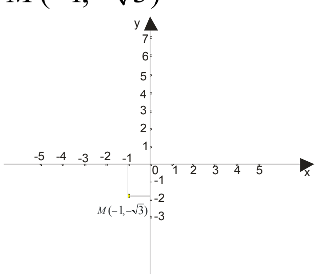
$$y`=\frac{2(x+1)}{(x^2+2)\sqrt{x^2+2}} \quad \rightarrow\rightarrow\rightarrow \quad y`=\frac{2(x+1)}{(x^2+2)^{\frac{3}{2}}}$$

y`=0 for x+1 = 0 , and then x = -1

<u>For x = -1</u>

$$y=\frac{-1-2}{\sqrt{1+2}}=\frac{-3}{\sqrt{3}}=\frac{-3}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\sqrt{3}$$

$M(-1,-\sqrt{3})$




| | -∞ -1 | ∞ |
|---|---|---|
| 2x+2 | − | + |
| y` | − | + |
| | ↘ | ↗ |

M is minimum point.

***convexity and concavity***

$$y=\frac{x-2}{\sqrt{x^2+2}}$$

$$y`=\frac{2(x+1)}{(x^2+2)^{\frac{3}{2}}}$$

$$y``=2\frac{(x+1)`\cdot(x^2+2)^{\frac{3}{2}}-((x^2+2)^{\frac{3}{2}})`(x+1)}{((x^2+2)^{\frac{3}{2}})^2}$$

$$y``=2\frac{1\cdot(x^2+2)^{\frac{3}{2}}-\frac{3}{2}(x^2+2)^{\frac{3}{2}-1}(x^2+2)`\cdot(x+1)}{(x^2+2)^3}$$

$$y``=2\frac{(x^2+2)^{\frac{3}{2}}-\frac{3}{\cancel{2}}(x^2+2)^{\frac{1}{2}}\cdot\cancel{2}x\cdot(x+1)}{(x^2+2)^3}$$

$$y``=2\frac{(x^2+2)^{\frac{3}{2}}-3(x^2+2)^{\frac{1}{2}}\cdot x\cdot(x+1)}{(x^2+2)^3}$$

$$y``=2\frac{\cancel{(x^2+2)^{\frac{1}{2}}}[x^2+2-3x\cdot(x+1)]}{(x^2+2)^{\cancel{3}}}$$

$$y``=2\frac{x^2+2-3x^2-3x}{(x^2+2)^{\frac{5}{2}}}$$

$$y``=2\frac{-2x^2-3x+2}{(x^2+2)^{\frac{5}{2}}}$$

$$y``=0$$

$$-2x^2-3x+2=0\rightarrow x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\rightarrow x_1=-2\wedge x_2=\frac{1}{2}$$

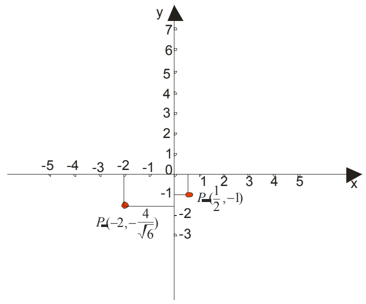
For $x_1=-2\rightarrow y_1=\frac{-4}{\sqrt{6}}$

For $x_2=\frac{1}{2}\rightarrow y_1=-1$

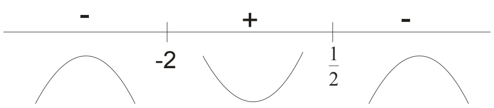
We have two points:

$\mathrm{P}_1(-2,\frac{-4}{\sqrt{6}})$

$P_2(\frac{1}{2},-1)$




Sign of second derivate, again, depends only on the terms in the numerator $-2x^2 - 3x + 2$.




***Asymptote function (behavior functions at the ends of the field definition)***

As we have already concluded, the function has no vertical asymptote!

<u>Horizontal asymptote</u>

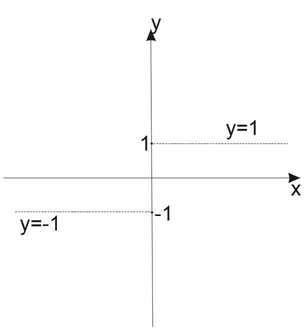
$\lim\limits_{x\to\pm\infty} \frac{x-2}{\sqrt{x^2+2}} = \lim\limits_{x\to\pm\infty} \frac{x-2}{\sqrt{x^2(1+\frac{2}{x^2})}} = \lim\limits_{x\to\pm\infty} \frac{x-2}{|x|\sqrt{(1+\frac{2}{x^2})}}$ Watch out! As we get down absolute value , we must separate limit for the + and - infinity!

$$\lim_{x\to+\infty} \frac{x-2}{x\sqrt{(1+\frac{2}{x^2})}} = 1$$

$$\lim_{x\to-\infty} \frac{x-2}{-x\sqrt{(1+\frac{2}{x^2})}} = -1$$

If x approaches + infinite horizontal asymptote is $y = 1$

IF x approaches - infinity horizontal asymptote is $y = -1$



The final graph is: