1. Examine function and draw a graph : $y=\ln\dfrac{x-2}{x+1}$

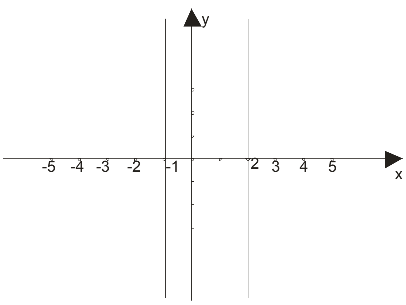
Domain

All behind ln must be greater than 0

$\dfrac{x-2}{x+1}>0$ We use a table …

	$-\infty$ … -1	-1 … 2	2 … ∞
x-2	−	−	+
x+1	−	+	+
$\dfrac{x-2}{x+1}$	+	−	+

Domain is $x\in(-\infty,-1)\cup(2,\infty)$. This tells us that there is no function between -1 and 2, the diagram is:



Zero function

To remind you: $\ln\Theta=0\leftrightarrow\Theta=1$

$y=0$

$\dfrac{x-2}{x+1}=1$

$x-2=x+1$

$-2=1$

So, there is no zero, which tells us that the function does not cut x axis

Sign function

Again a reminder:

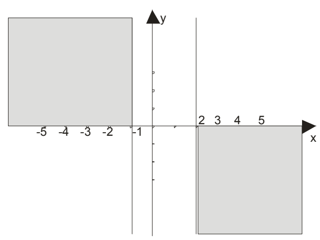
$\ln\Theta>0\leftrightarrow\Theta>1$

$\ln\Theta<0\leftrightarrow<0<\Theta<1$

So:

$$y > 0$$
$$\frac{x-2}{x+1} > 1$$
$$\frac{x-2}{x+1} - 1 > 0$$
$$\frac{x-2-1(x+1)}{x+1} > 0$$
$$\frac{x-2-x-1}{x+1} > 0$$
$$\frac{-3}{x+1} > 0 \quad ..../* (-1)$$
$$\frac{3}{x+1} < 0$$
$$x+1 < 0$$
$$\boxed{x < -1}$$

y > 0 for x < -1 and y < 0 for x > 2



Parity

The function is neither even nor odd. It is clear to us from the domain is ... If you really must, then:

$$f(-x) = \ln\frac{-x-2}{-x+1} \neq f(x)$$

Extreme values (max and min) and monotonic function (increasing and decreasing)

$$y = \ln\frac{x-2}{x+1}$$
$$y` = \frac{1}{\frac{x-2}{x+1}}\left(\frac{x-2}{x+1}\right)` = \frac{x+1}{x-2}\cdot\frac{(x-2)`(x+1)-(x+1)`(x-2)}{(x+1)^2} = \frac{\cancel{x+1}}{x-2}\cdot\frac{1(x+1)-1(x-2)}{(x+1)^{\cancel{2}}} = \frac{x+1-x+2}{(x-2)(x+1)}$$
$$y` = \frac{3}{(x-2)(x+1)}$$

y` = 0 for 3=0 , so we conclude that there is no extreme values.

Still thinking of which depends on the sign of the first derivate? Since (x-2)(x+1).

	$-\infty$ -1	2	∞
x-2	—	—	+
x+1	—	+	+
y`	+ ↗	— no function	+ ↗

This indicates that the function is **always** monotone **increasing.**

convexity and concavity

$$y = \ln\frac{x-2}{x+1}$$

$$y` = \frac{3}{(x-2)(x+1)} \qquad \text{we know that } \left(\frac{1}{\otimes}\right)` = -\frac{1}{\otimes^2}\cdot\otimes`$$

$$y`` = -\frac{3}{(x-2)^2(x+1)^2}[(x-2)(x+1)]`$$

$$y`` = -\frac{3}{(x-2)^2(x+1)^2}[1(x+1)+1(x-2)]$$

$$y`` = -\frac{3}{(x-2)^2(x+1)^2}(2x-1)$$

$$y`` = \frac{3(1-2x)}{(x-2)^2(x+1)^2}$$

y`` = 0 for 1-2x = 0 , so $x = \frac{1}{2}$, **but** beware, this point **does not belong to** the domain!

$$y`` > 0 \rightarrow 1-2x > 0 \rightarrow x < \frac{1}{2} \rightarrow x < -1$$

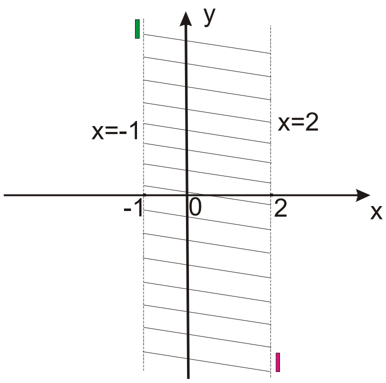
$$y`` < 0 \rightarrow 1-2x < 0 \rightarrow x > \frac{1}{2} \rightarrow x > 2$$

Asymptote function (behavior functions at the ends of the field definition)

Vertical asymptote

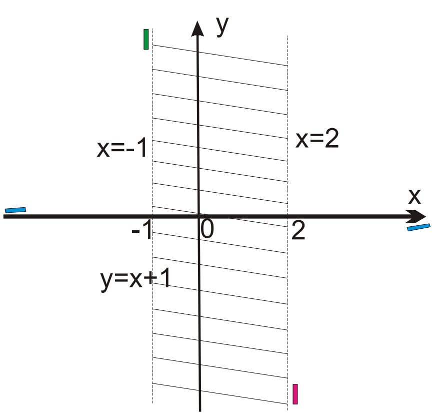
$$\lim_{x\to 2+\varepsilon} \ln\frac{x-2}{x+1} = \ln\frac{2+\varepsilon-2}{2+1} = \ln 0 = -\infty \text{ (red line)}$$

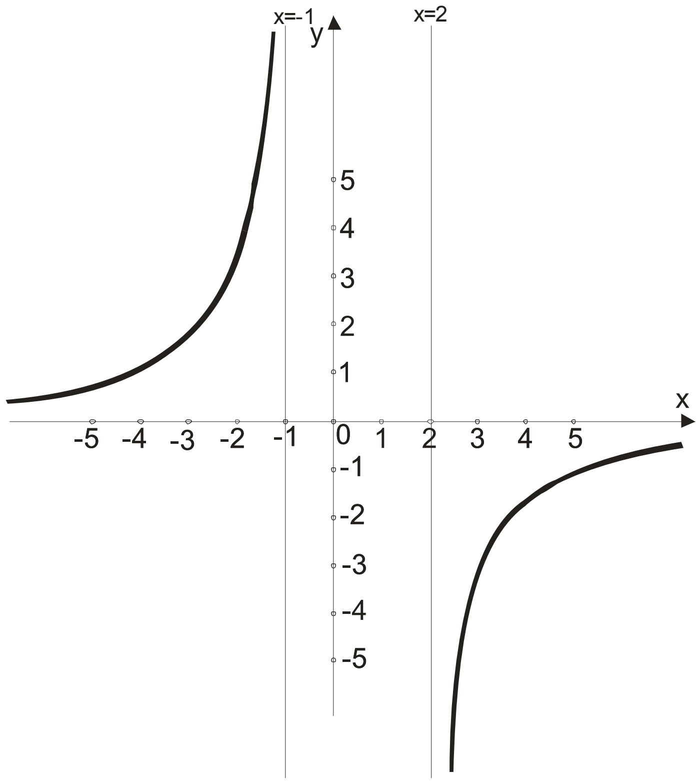
$$\lim_{x\to -1-\varepsilon} \ln\frac{x-2}{x+1} = \ln\frac{-1-2}{-1-\varepsilon+1} = \ln\frac{-3}{-\varepsilon} = \ln\infty = \infty \text{ (green line)}$$

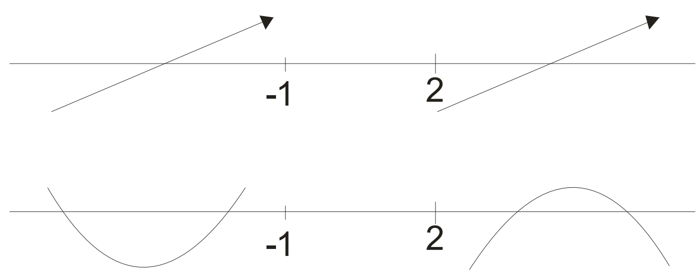


Horizontal asymptote

$$\lim_{x\to\pm\infty} \ln\frac{x-2}{x+1} = \ln\lim_{x\to\pm\infty}\frac{x-2}{x+1} = \ln 1 = 0$$ So: $y = 0$ (x-axis) is a horizontal asymptote. (Blue lines)







2. Examine function and draw a graph : $y = \dfrac{1+\ln x}{1-\ln x}$

Domain

All behind *ln* must be larger than 0, so there is x > 0.

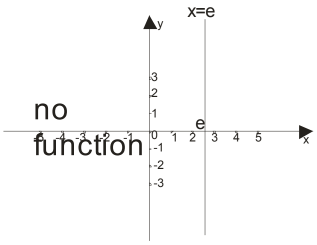
As we have a fraction, all in denominator must be different from 0.

$$1-\ln x \neq 0$$
$$\ln x \neq 1$$
$$x \neq e$$

domain is :

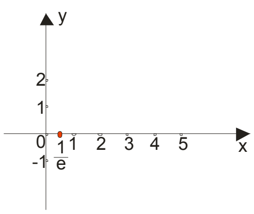
$$x \in (0,e) \cup (e,\infty)$$

On the diagram, it would look like this:



Zero function

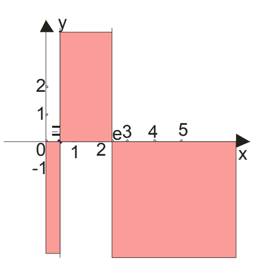
$$y = 0 \rightarrow 1 + \ln x = 0 \rightarrow \ln x = -1 \rightarrow x = e^{-1} = \frac{1}{e}$$



Sign function

	0	$\frac{1}{e}=e^{-1}$	e	∞
1+lnx	−	+	+	
1-lnx	+	+	−	
y	−	+	−	

On the diagram, it would look like this:



The function is only in marked areas.

Parity

The function is neither even nor odd. Why?

And it makes no sense to require f (-x) because function is not defined for x <0

Extreme values (max and min) and monotonic function (increasing and decreasing)

$$y=\frac{1+\ln x}{1-\ln x}$$

$$y`=\frac{(1+\ln x)`(1-\ln x)-(1-\ln x)`(1+\ln x)}{(1-\ln x)^2}$$

$$y`=\frac{\frac{1}{x}(1-\ln x)-(-\frac{1}{x})(1+\ln x)}{(1-\ln x)^2}$$

$$y`=\frac{\frac{1}{x}[(1-\ln x)+(1+\ln x)]}{(1-\ln x)^2}=\frac{1-\cancel{\ln x}+1+\cancel{\ln x}}{x\cdot(1-\ln x)^2}=\frac{2}{x\cdot(1-\ln x)^2}$$

As $2\neq 0$ function has not extreme values.

Think further: x > 0 always and $(1-\ln x)^2>0$, so y`>0 for all x .
The function is **always** monotone **increasing.**

convexity and concavity

$$y=\frac{1+\ln x}{1-\ln x}$$

$$y`=\frac{2}{x\cdot(1-\ln x)^2}$$

$$y``=-\frac{2}{x^2\cdot(1-\ln x)^4}\cdot[x\cdot(1-\ln x)^2]`$$

$$y``=-\frac{2}{x^2\cdot(1-\ln x)^4}\cdot[x`\cdot(1-\ln x)^2+((1-\ln x)^2)`x]$$

$$y``=-\frac{2}{x^2\cdot(1-\ln x)^4}\cdot[1\cdot(1-\ln x)^2+2(1-\ln x)(-\frac{1}{x})x]$$

$$y``=-\frac{2}{x^2\cdot(1-\ln x)^4}\cdot[1\cdot(1-\ln x)^2+2(1-\ln x)(-1)]=-\frac{2}{x^2\cdot(1-\ln x)^4}\cdot[(1-\ln x)^2-2(1-\ln x)]$$

$$y``=-\frac{2}{x^2\cdot(1-\ln x)^{\cancel{4}}}\cdot\cancel{(1-\ln x)}[1-\ln x-2]=-\frac{2}{x^2\cdot(1-\ln x)^3}\cdot[-\ln x-1]$$

$$y``=\frac{2(1+\ln x)}{x^2\cdot(1-\ln x)^3}$$

$$y``=0\rightarrow 1+\ln x=0\rightarrow \ln x=-1\rightarrow x=e^{-1}=\frac{1}{e}$$

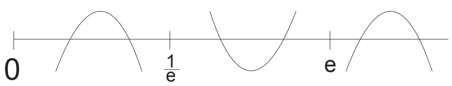
$$y=\frac{1+\ln e^{-1}}{1-\ln e^{-1}}=\frac{1-1}{1+1}=0$$

We find point $P(\frac{1}{e},0)$

What factors influence the sign of the second derivate?

1+lnx and 1-lnx. Idemo u tablicu… Go to the table ...

	0	$\frac{1}{e}=e^{-1}$	e ∞
1+lnx	−	+	+
1-lnx	+	+	−
y``	−	+	−

if we draw this: 

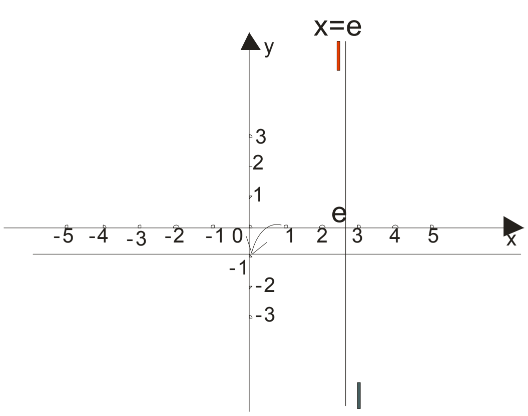
Asymptote function (behavior functions at the ends of the field definition)

<u>Vertical asymptote</u>

$$\lim_{x\to o+\varepsilon}\frac{1+\ln x}{1-\ln x}=\frac{1+\ln(0+\varepsilon)}{1-\ln(0+\varepsilon)}=\frac{\infty}{\infty}=l'H\hat{o}pital=\lim_{x\to o+\varepsilon}\frac{\frac{1}{x}}{-\frac{1}{x}}=-1 \text{ (arrow)}$$

$$\lim_{x\to e+\varepsilon}\frac{1+\ln x}{1-\ln x}=\frac{1+\ln(e+\varepsilon)}{1-\ln(e+\varepsilon)}=\frac{2}{-\varepsilon}=-\infty \text{ (green line)}$$

$$\lim_{x\to e-\varepsilon}\frac{1+\ln x}{1-\ln x}=\frac{1+\ln(e-\varepsilon)}{1-\ln(e-\varepsilon)}=\frac{2}{+\varepsilon}=+\infty \text{ (red line)}$$



<u>Horizontal asymptote</u>

$$\lim_{x\to\infty}\frac{1+\ln x}{1-\ln x}=\frac{\infty}{-\infty}=l'H\hat{o}pital=\lim_{x\to\infty}\frac{\frac{1}{x}}{-\frac{1}{x}}=-1$$

$y=-1$

final graph: