## GRAPHICAL RESOLVING SYSTEM

The most common type of task is one in which there is a **quadratic function** $y = ax^2 + bx + c$ and **linear function** $y = kx + n$.

Our advice is to first solve the system analytically and then to draw graphics. If you draw the graph first it may happen that **you can not determine the coordinates of** section.

Here are a few examples:

Example 1.

**Graphic solve the system of equations:**

$$x^2 - 2x + y + 4 = 0$$
$$x + y + 2 = 0$$

**Solution:**

First, we express $y$ in either equation and solve the system analytically:

$$x^2 - 2x + y + 4 = 0 \rightarrow \boxed{y = -x^2 + 2x - 4}$$
$$x + y + 2 = 0 \rightarrow \boxed{y = -x - 2}$$

Now set up a single equation,by comparing the left sides of the two equality (the rights are the same)

$$-x^2 + 2x - 4 = -x - 2$$
$$-x^2 + 2x - 4 + x + 2 = 0$$
$$-x^2 + 3x - 2 = 0$$
$$a = -1; b = 3; c = -2$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4\cdot(-1)\cdot(-2)}}{2\cdot(-1)} = \frac{-3 \pm \sqrt{1}}{-2} = \frac{-3 \pm 1}{-2}$$
$$x_1 = \frac{-3+1}{-2} = \frac{-2}{-2} \rightarrow \boxed{x_1 = 1}$$
$$x_2 = \frac{-3-1}{-2} = \frac{-4}{-2} \rightarrow \boxed{x_2 = 2}$$

Now, these values back into equation $y = -x - 2$, to find the $y$ coordinates

For $x_1 = 1$ is $y_1 = -1 - 2 \rightarrow \boxed{y_1 = -3}$ so, one solution is point ( 1, - 3)

For $x_2 = 2$ is $y_1 = -2 - 2 \rightarrow \boxed{y_1 = -4}$ and the second solution is point ( 2,- 4)

Now we can draw graphs, but in the same coordinate system.

Of course, it is easier to draw the line ... we will take two points, say x = 0 and find y, then we take y = 0 and find x

For $y = -x - 2$ we have

| x | 0 | -2 |
|---|---|---|
| y | -2 | 0 |

For square function we will examine only the necessary :

$y = -x^2 + 2x - 4$

**Zero function:**

$$-x^2 + 2x - 4 = 0$$
$$a = -1; b = 2; c = -4$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-1)(-4)}}{2(-1)} = \frac{-2 \pm \sqrt{-12}}{-12}$$

From here we conclude that we have no real solutions, that the graph of quadratic functions nowhere cuts x axis.

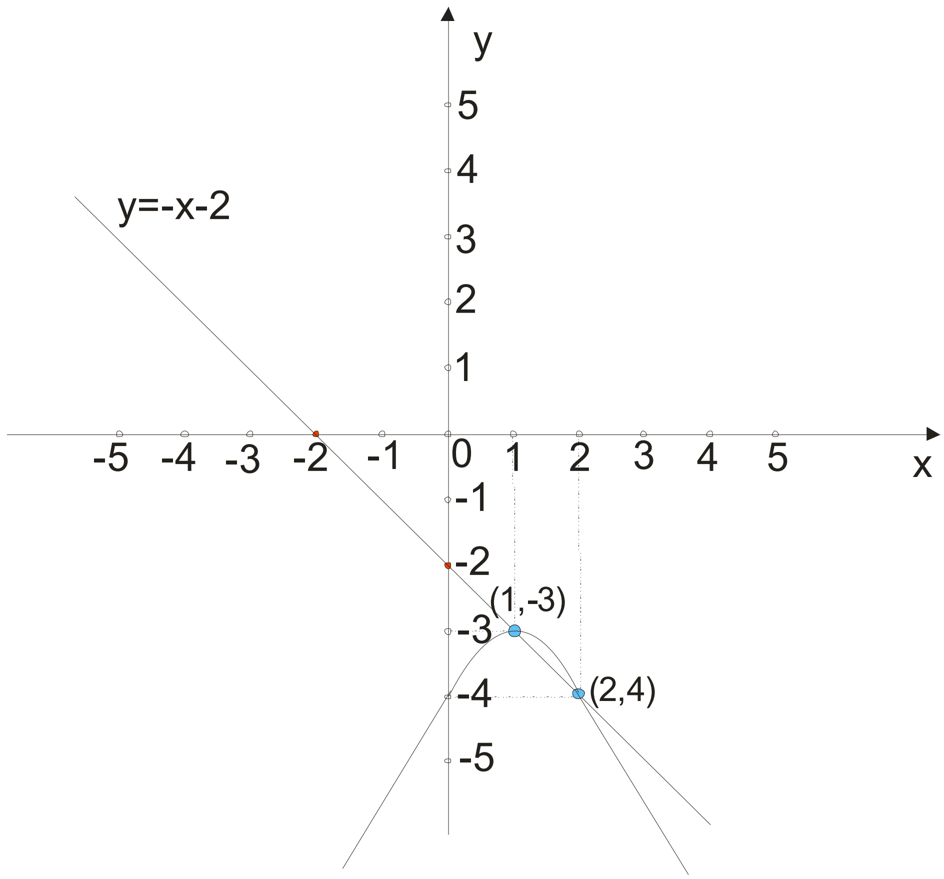
**The intersection of the y-line**

To remind, intersection with y axis is c , in this case is $c = -4$

**Max or min:**

$$T(\alpha, \beta)$$
$$\alpha = -\frac{b}{2a} = -\frac{2}{2 \cdot (-1)} = 1$$
$$\beta = -\frac{D}{4a} = -\frac{b^2 - 4ac}{4a} = -\frac{-12}{4 \cdot (-1)} = -3$$
$$T(1, -3)$$

Now we can draw a graphic:




We see that the graphics and analytical solutions are same.

Example 2.

**Graphic solve the system of equations:**

$$y=x^2-4x+3$$
$$y=2x-6$$

**Solution:**

First, to solve analytically:

$$y=x^2-4x+3$$
$$\underline{y=2x-6}$$
$$x^2-4x+3=2x-6$$
$$x^2-4x+3-2x+6=0$$
$$x^2-6x+9=0 \rightarrow (x-3)^2=0 \rightarrow \boxed{x_1=x_2=3} \rightarrow y=2\cdot 3-6 \rightarrow \boxed{y_1=y_2=0}$$

Therefore, there is only **one** solution of this system, the point (3.0). It tells us that the graphics of parabola and line are cut in only **one point**.

**For line** $y = 2x - 6$ is

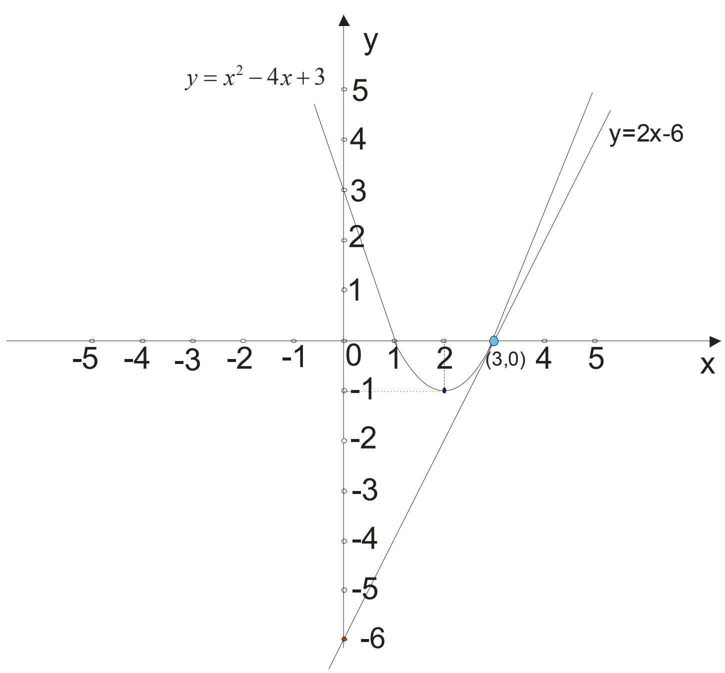
| x | 0 | -3 |
|---|---|---|
| y | -6 | 0 |

**For parable** $y = x^2 - 4x + 3$:

$$x^2 - 4x + 3 = 0$$
$$a = 1; b = -4; c = 3$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2} = \frac{4 \pm 2}{2}$$
$$x_1 = 3; x_2 = 1$$

Intersection with the y axis is *c,* in this case is $c = 3$

$$T(\alpha, \beta)$$
$$\alpha = -\frac{b}{2a} = -\frac{-4}{2 \cdot 1} = 2$$
$$\beta = -\frac{D}{4a} = -\frac{b^2 - 4ac}{4a} = -\frac{4}{4 \cdot 1} = -1$$
$$\boxed{T(2, -1)}$$



Example 3.

**Graphic solve the system of equations:**

$y = x^2$

$y = x - 1$

**Solution:**

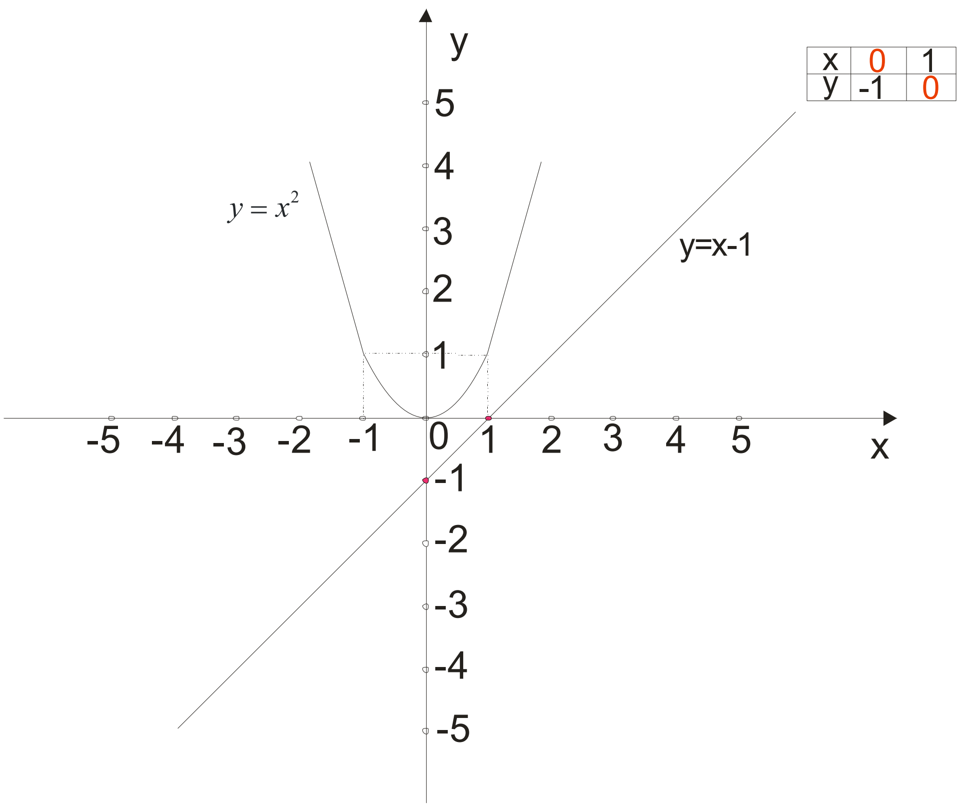
$y = x^2$

$y = x - 1$

$x^2 = x - 1$

$x^2 - x + 1 = 0 \rightarrow D = b^2 - 4ac = 1 - 4 = -3 \rightarrow \boxed{D < 0}$

The system has no real solutions. So, **graphics are not cutting!**




**Conclusion:**

**When we have to solve the system graphically it is possible to have two intersecting points (example 1), to intersect in one point (Example 2) or no section (example 3)**

Here are a few examples where we have not a linear function .

Example 4.

**Graphic solve the system of equations:**

$xy = 12$
$x + y = 7$

**Solution:**

As always, first solve the system analytically:

$$\begin{aligned}
&xy = 12 \\
&\underline{x + y = 7} \\
&x + y = 7 \rightarrow y = 7 - x \rightarrow \text{ substituting in } xy = 12 \\
&x(7 - x) = 12 \\
&7x - x^2 - 12 = 0 \\
&x^2 - 7x + 12 = 0 \rightarrow x_{1,2} = \frac{7 \pm \sqrt{1}}{2} \rightarrow x_1 = 4 \wedge x_2 = 3 \\
&x_1 = 4 \rightarrow y_1 = 7 - x_1 \rightarrow y_1 = 3 \rightarrow \boxed{(4,3)} \\
&x_2 = 3 \rightarrow y_2 = 7 - x_2 \rightarrow y_2 = 4 \rightarrow \boxed{(3,4)}
\end{aligned}$$

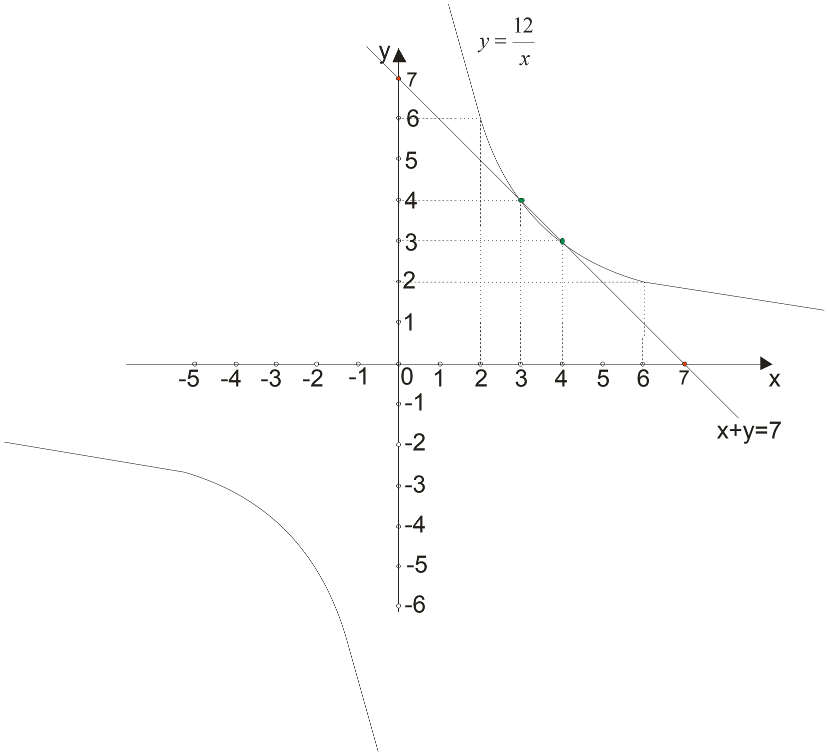
For the line , as always, we take two points:

| x | 0 | 7 |
|---|---|---|
| y | 7 | 0 |

For hyperbola $y = \frac{12}{x}$ we'll take a few points, and if you remember from before, it will belong to the first and third quadrant:

| x | -4 | -3 | -2 | -1 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| y | -3 | -4 | -6 | -12 | 12 | 6 | 4 | 3 |

Graph will be:




Example 5.

**Graphic solve the system of equations:**

$$y = x^2 - 4x + 4$$
$$y = -x^2 + 3x - 2$$

**Solution:**

$$y = x^2 - 4x + 4$$
$$\underline{y = -x^2 + 3x - 2}$$
$$x^2 - 4x + 4 = -x^2 + 3x - 2$$
$$x^2 - 4x + 4 + x^2 - 3x + 2 = 0$$
$$2x^2 - 7x + 6 = 0 \rightarrow x_1 = 2 \wedge x_2 = \frac{3}{2}$$

Now, these values replace in any of the two equations (for example, in the first)

$$y = x^2 - 4x + 4$$
$$x_1 = 2 \wedge x_2 = \frac{3}{2}$$
$$x_1 = 2 \rightarrow y_1 = 2^2 - 4\cdot 2 + 4 = 0 \rightarrow \boxed{(2,0)}$$
$$x_2 = \frac{3}{2} \rightarrow y_2 = \left(\frac{3}{2}\right)^2 - 4\cdot\frac{3}{2} + 4 = \frac{1}{4} \rightarrow \boxed{(\frac{3}{2},\frac{1}{4})}$$

**We got the point of intersection.**

As already known procedure to examine the course of two given quadratic functions :