REŠENJA TESTA IZ MATEMATIKE NA KLASIFIKACIONOM ISPITU 2008. GODINE

1. Uprostiti izraz $I = \dfrac{2}{x-1} - \dfrac{x^2+1}{x^2-1} + \dfrac{x}{x+1}$.

$$I = \frac{2}{x-1} - \frac{x^2+1}{(x-1)(x+1)} + \frac{x}{x+1} = \frac{2(x+1)-(x^2+1)+x(x-1)}{(x-1)(x+1)}$$
$$= \frac{2x+2-x^2-1+x^2-x}{(x-1)(x+1)} = \frac{x+1}{(x-1)(x+1)} = \frac{1}{x-1}, x \neq \pm 1.$$

2. Rastaviti na faktore polinom $P(x) = x^5 - x^3 - x^2 + 1$.

$$P(x) = x^3(x^2-1) - (x^2-1) = (x^2-1)(x^3-1)$$
$$= (x-1)(x+1)(x-1)(x^2+x+1) = (x+1)(x-1)^2(x^2+x+1).$$

3. Odrediti NZD za polinome $x^2 - y^2$ i $x^2 - 2xy + y^2$.

$x^2 - y^2 = (x-y)(x+y); \quad x^2 - 2xy + y^2 = (x-y)^2;$

$$NZD = x - y.$$

4. Rešiti jednačinu $\dfrac{6x-1}{2+x} = 3$.

Za $x \neq -2$, je $\dfrac{6x-1}{2+x} = 3 \iff 6x - 1 = 3(2+x) \iff 3x = 7 \iff x = \dfrac{7}{3}$.

5. Odrediti konstante A i B tako da važi $\dfrac{A}{x-3} + \dfrac{B}{2x+4} = \dfrac{2x+1}{(x-3)(2x+4)}$.

$$\frac{A}{x-3} + \frac{B}{2x+4} = \frac{2x+1}{(x-3)(2x+4)} \iff \frac{A(2x+4)}{x-3} + \frac{B(x-3)}{2x+4} = \frac{2x+1}{(x-3)(2x+4)}$$
$$\iff \frac{2Ax+4A+Bx-3B}{(x-3)(2x+4)} = \frac{2x+1}{(x-3)(2x+4)} \Rightarrow$$

$$(2A+B)x + 4A - 3B = 2x + 1 \iff 2A + B = 2 \wedge 4A - 3B = 1 \iff A = \frac{7}{10} \wedge B = \frac{3}{5}.$$

6. Rešiti sistem jednačina $\dfrac{y}{x} = \dfrac{1}{2} \quad \wedge \quad 2x + y = 15$.

$$\frac{y}{x} = \frac{1}{2} \wedge 2x + y = 15 \iff x = 2y \wedge 2x + y = 15 \iff x = 2y \wedge 2(2y) + y = 15$$
$$\iff x = 2y \ \wedge \ 5y = 15 \iff y = 3 \ \wedge \ x = 6.$$

7. Rešiti jednačinu $x^2 + 2x - 15 = 0$.

$$x_{1,2} = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-15)}}{2 \cdot 1} \iff x_{1,2} = \frac{-2 \pm \sqrt{64}}{2} \iff x_{1,2} = \frac{-2 \pm 8}{2} \iff x_1 = 3 \vee x_2 = -5.$$

8. Za koju vrednost parametra $m \in \mathbb{R}$ kvadratna jednačina $mx^2 + 6x + 3 = 0$ ima dvostruko rešenje?

$D = 0 \iff 36 - 12m = 0 \iff m = 3.$

9. Rešiti nejednačinu $x(x-1) < 6$.

$x(x-1) < 6 \iff x^2 - x - 6 < 0 \iff (x-3)(x+2) < 0 \iff$
$(x - 3 < 0 \land x + 2 > 0) \lor (x - 3 > 0 \land x + 2 < 0) \iff x \in (-2, 3).$

10. Odrediti oblast definisanosti, nulu, znak i nacrtati grafik funkcije $y = \ln x$.

$D = \{x \in \mathbb{R} \mid x > 0\} = (0, +\infty) = \mathbb{R}^+.$

$y = 0 \iff \ln(x) = 0 \iff x = e^0 \iff$
$x = 1$ - nula funkcije.
$A(1,0)$ i $B(e,1)$ - tačke koje pripadaju grafiku.

$y > 0$ za $x \in (1, +\infty)$ i $y < 0$ za $x \in (0,1)$.

11. Rešiti jednačinu $\dfrac{\ln x^2}{\ln(1-x)} = 2$.

$\dfrac{\ln x^2}{\ln(1-x)} = 2 \iff 2\ln x = 2\ln(1-x) \iff \ln x = \ln(1-x) \iff$
$x = 1 - x \iff 2x = 1 \iff x = \dfrac{1}{2}.$

12. Rastaviti na faktore polinom $81x^3 - 3$.

$81x^3 - 3 = 3(27x^3 - 1) = 3(3^3x^3 - 1) = 3((3x)^3 - 1^3) = 3(3x-1)(9x^2 + 3x + 1).$

13. Rešiti nejednačinu $(3x-4)(2-x) \geq 0$.

$(3x-4)(2-x) \geq 0 \iff (3x - 4 \geq 0 \land 2 - x \geq 0) \lor (3x - 4 \leq 0 \land 2 - x \leq 0) \iff$
$(x \geq \frac{4}{3} \land x \leq 2) \lor (x \leq \frac{4}{3} \land x \geq 2) \iff \frac{4}{3} \leq x \leq 2 \lor x \in \emptyset \iff x \in \left[\frac{4}{3}, 2\right].$

14. Rešiti jednačinu $\left(\dfrac{1}{8}\right)^{2x+1} = 2^{3x}$.

$\left(\dfrac{1}{8}\right)^{2x+1} = 2^{3x} \iff \left(\dfrac{1}{2^3}\right)^{2x+1} = 2^{3x} \iff (2^{-3})^{2x+1} = 2^{3x} \iff$
$2^{-3(2x+1)} = 2^{3x} \iff -6x - 3 = 3x \iff -3 = 9x \iff x = -\dfrac{1}{3}.$

15. Napisati kanonski oblik parabole $y = x^2 - 4x + 3$.

$y = 1\left(x + \dfrac{-4}{2 \cdot 1}\right)^2 + \dfrac{4 \cdot 1 \cdot 3 - (-4)^2}{4 \cdot 1} = (x-2)^2 - 1.$