Stepen i kvadratni koren

20. Израчунати:

А) $11\cdot 3^2-\frac{9^2}{3}$;

Б) $(3\cdot 2)^2-\frac{1}{2}\cdot 4^2$.

A)

$$11\cdot 3^2-\frac{9^2}{3}=11\cdot 9-\frac{81}{3}$$
$$=99-27$$
$$=72$$

B)

$$(3\cdot 2)^2-\frac{1}{2}\cdot 4^2=6^2-\frac{1}{2}\cdot 16$$
$$=36-8$$
$$=28$$

21. Израчунати $\left(2^3\cdot 3^2\right):6+\left(2^3-3^2\right)\cdot 10^2$.

$$(2^3\cdot 3^2):6+(2^3-3^2)\cdot 10^2=$$
$$(8\cdot 9):6+(8-9)\cdot 100=$$
$$72:6+(-1)\cdot 100=12-100=-88$$

22. Израчунати:

А) $\frac{5}{3^2}+\left(\frac{2}{3}\right)^3-\frac{4^2}{9}$;

Б) $\left(-\frac{3}{5}\right)^3\cdot(12-17)^2$.

A)

$$\frac{5}{3^2}+\left(\frac{2}{3}\right)^3-\frac{4^2}{9}=\frac{5^{(\cdot 3}}{9}+\frac{8}{27}-\frac{16^{(\cdot 3}}{9}=$$
$$=\frac{15+8-48}{27}=-\frac{25}{27}$$

B)

$$\left(-\frac{3}{5}\right)^3\cdot(12-17)^2=-\frac{27}{125}\cdot(-5)^2$$
$$=-\frac{27}{125}\cdot 25$$
$$=-\frac{27}{5}$$

23. Израчунати $\dfrac{2^3 \cdot 4^2}{16^2 : 8}$.

$$\frac{2^3 \cdot 4^2}{16^2 : 8} =$$

Pazi, ovde ne možemo odmah upotrebiti pravila za stepenovanje jer osnove nisu iste! Zato ćemo uraditi:

$$4 = 2^2,\ 16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4,\ 8 = 2 \cdot 2 \cdot 2 = 2^3$$

Vratimo se u zadatak:

$$\frac{2^3 \cdot 4^2}{16^2 : 8} = \frac{2^3 \cdot (2^2)^2}{(2^4)^2 : 2^3} = \frac{2^3 \cdot 2^4}{2^8 : 2^3} = \frac{2^{3+4}}{2^{8-3}} = \frac{2^7}{2^5} = 2^{7-5} = 2^2 = 4$$

24. Израчунати $2^5 \cdot \dfrac{4^3 \cdot 8}{4^2 \cdot 2^6} - 2^3 \cdot \dfrac{16^2 \cdot 2^4}{2^3 \cdot 8^3}$.

$$2^5 \cdot \frac{4^3 \cdot 8}{4^2 \cdot 2^6} - 2^3 \cdot \frac{16^2 \cdot 4^2}{2^3 \cdot 8^3} =$$

$$2^5 \cdot \frac{(2^2)^3 \cdot 2^3}{(2^2)^2 \cdot 2^6} - 2^3 \cdot \frac{(2^4)^2 \cdot 2^4}{2^3 \cdot (2^3)^3} =$$

$$\frac{2^5}{1} \cdot \frac{\cancel{2^6} \cdot 2^3}{2^4 \cdot \cancel{2^6}} - \frac{\cancel{2^3}}{1} \cdot \frac{2^8 \cdot 2^4}{\cancel{2^3} \cdot 2^9} =$$

$$\frac{2^5 \cdot 2^3}{2^4} - \frac{2^8 \cdot 2^4}{2^9} =$$

$$\frac{2^8}{2^4} - \frac{2^{12}}{2^9} = 2^4 - 2^3 = 16 - 8 = 8$$

25. Израчунати $\dfrac{81^2 \cdot 3^2 : 27}{3^5 \cdot 9}$.

$$\frac{81^2 \cdot 3^2 : 27}{3^5 \cdot 9} =$$

Opet sličan problem, moramo ‘’napraviti’’ iste osnove!

$$81 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4,\ 27 = 3 \cdot 3 \cdot 3 = 3^4,\ 9 = 3^2$$

Sada je:

$$\frac{81^2 \cdot 3^2 : 27}{3^5 \cdot 9} = \frac{(3^4)^2 \cdot 3^2 : 3^3}{3^5 \cdot 3^2} = \frac{3^8 \cdot 3^2 : 3^3}{3^7} = \frac{3^{8+2-3}}{3^7} = \frac{3^7}{3^7} = 1$$

26. **Израчунати** $3^4+36^6+15^7-6^{12}-3^7\cdot 5^7$.

$3^4+36^6+15^7-6^{12}-3^7\cdot 5^7=$

$3^4+(6^2)^6+15^7-6^{12}-(3\cdot 5)^7=$

$3^4+6^{12}+15^7-6^{12}-15^7=$ (potiru se!)

$=3^4=81$

27. **Израчунати:**

А) $\dfrac{3^7+3^5}{3^7-3^5}$;

Б) $\dfrac{2^8-2^5}{2^8+2^5}$.

A)

U ovom zadatku ćemo upotrebiti distributivni zakon, odnosno izvući ćemo 'zajednički' ispred zagrade!

A) $\dfrac{3^7+3^5}{3^7-3^5}=\dfrac{3^5\cdot 3^2+3^5}{3^5 3^2-3^5}=\dfrac{3^5(3^2+1)}{3^5(3^2+1)}=\dfrac{9+1}{9-1}=\dfrac{10}{8}=\dfrac{5}{4}$

B) $\dfrac{2^8-2^5}{2^8+2^5}=\dfrac{2^5\cdot 2^3-2^5}{2^5\cdot 2^3+2^5}=\dfrac{2^5(2^3-1)}{2^5(2^3+1)}=\dfrac{8-1}{8+1}=\dfrac{7}{9}$

28. **Упростити изразе:**

А) $12a^6b^3c:(-3a^2b)\ \ (a\neq 0, b\neq 0)$;

Б) $-4a^3b^3c:(2ab^2c)\ \ (a\neq 0, b\neq 0, c\neq 0)$.

A) $12a^6b^3c:(-3a^2b)=$

Kako ovde razmišljati?

Uočiti brojke, $a-ove$ i... $b-ove$

$12a^6b^3c:(-3a^2b)=$

$=[12:(-3)]\cdot[a^6:a^2]\cdot[b^3:b]\cdot c$

$=-4\cdot a^4\cdot b^2\cdot c$

B)

$-4a^3b^3c:(2ab^2c)=$

$=[4:2]\cdot[a^3:a]\cdot[b^3:b^2]\cdot[c:c]$

$=-2\cdot a^2\cdot b^1\cdot 1$

$=-2a^2b$

30. Упростити израз $\left(\frac{(2a^3)^2}{2a^4b^3}\right)^3$ $(a \neq 0,\ b \neq 0)$.

$$\left(\frac{(2a^3)^2}{2a^4b^3}\right)^3 = \left(\frac{4a^6}{2a^4b^3}\right)^3 = \left(\frac{2a^2}{b^3}\right)^3 = \frac{8a^6}{b^9}$$

31. Израчунати вредност израза $\frac{(-2)^3}{-2^2} - \frac{(-2)^2}{-2^3}$.

$$\frac{(-2)^3}{-2^2} - \frac{(-2)^2}{-2^3} = \frac{-8}{-4} - \frac{4}{-8} = 2 + \frac{1}{2} = 2\frac{1}{2} = \frac{5}{2}$$

32. Упростити изразе:

А) $(-a)^2 \cdot (-a)^3 \cdot (-a)^5$;

Б) $(-x)^2 \cdot (-x^2) \cdot (-x^3) \cdot (-x)^3$.

A) $(-a)^2 \cdot (-a)^3 \cdot (-a)^5 = (-a)^{2+3+5} = (-a)^{10} = a^{10}$

B) $(-x)^2 \cdot (-x^2) \cdot (-x^3) \cdot (-x)^3 =$ pažljivo

$= x^2 \cdot (-x^2) \cdot (-x^3) \cdot (-x^3) = [$ ima tri minusa koji daju opet $-$ $]$

$= -x^{2+2+3+3} = -x^{10}$

33. Упростити изразе:

А) $\frac{3^{6n} \cdot 3^{2n+1}}{3^{3n} \cdot 3^{5n}}$;

Б) $\frac{8^{2n-1}}{2^{6n-5}}$.

A) $\frac{3^{6n} \cdot 3^{2n+1}}{3^{3n} \cdot 3^{5n}} = \frac{3^{6n+2n+1}}{3^{3n+5n}} = \frac{3^{8n+1}}{3^{8n}} = 3^{8n+1-8n} = 3^1 = 3$

B) $\frac{8^{2n-1}}{2^{6n-5}} = \frac{(2^3)^{2n-1}}{2^{6n-5}} = \frac{2^{6n-3}}{2^{6n-5}} =$ pazi na minus!!!

$= 2^{6n-3-(6n-5)} = 2^{6n-3-6n+5} = 2^2 = 4$

34. Израчунати вредност израза

А) $\sqrt{1-\frac{9}{25}}+\sqrt{1+\frac{39}{25}}$;

Б) $\sqrt{\frac{1}{4}}-\sqrt{\frac{1}{(-2)^2}}$.

A) $5\sqrt{2}+3\sqrt{8}-\sqrt{50}-\sqrt{98}=$ [pazi, moramo najpre da sredimo izraze unutar korena]

$$\sqrt{8}=\sqrt{4\cdot 2}=\sqrt{4}\cdot\sqrt{2}=2\sqrt{2}$$
$$\sqrt{50}=\sqrt{25\cdot 2}=\sqrt{25}\cdot\sqrt{2}=5\sqrt{2}$$
$$\sqrt{98}=\sqrt{49\cdot 2}=\sqrt{49}\cdot\sqrt{2}=7\sqrt{2}$$

Vratimo se sad na zadatak:

$$\cancel{5\sqrt{2}}+3\cdot 2\sqrt{2}-\cancel{5\sqrt{2}}-7\sqrt{2}=6\sqrt{2}-7\sqrt{2}=-\sqrt{2}$$

35. Упростити израз $5\sqrt{2}+3\sqrt{8}-\sqrt{50}-\sqrt{98}$.

A) $2\sqrt{2}+\sqrt{72}-3\sqrt{8}=2\sqrt{2}+\sqrt{36\cdot 2}-3\cdot\sqrt{4\cdot 2}$

$$=2\sqrt{2}+6\sqrt{2}-3\cdot 2\sqrt{2}$$
$$=2\sqrt{2}+6\sqrt{2}-6\sqrt{2}$$
$$=2\sqrt{2}$$

B) $-3\sqrt{(-2)^2}+(2\sqrt{3})^2-\left(4\sqrt{5}\right)^2=$

$$-3\sqrt{4}+2^2\cdot\sqrt{3}^2-4^2\sqrt{5}^2=$$
$$-3\cdot 2+4\cdot 3-16\cdot 5=-6+12-80=-74$$

36. Упростити изразе:

А) $2\sqrt{2}+\sqrt{72}-3\sqrt{8}$;

Б) $-3\sqrt{(-2)^2}+\left(2\sqrt{3}\right)^2-\left(4\sqrt{5}\right)^2$.

A)

$$2\sqrt{2}+\sqrt{72}-3\sqrt{8}=$$
$$2\sqrt{2}+\sqrt{36\cdot 2}-3\sqrt{4\cdot 2}=$$
$$2\sqrt{2}+6\sqrt{2}-3\cdot 2\sqrt{2}=$$
$$2\sqrt{2}\cancel{+6\sqrt{2}}\cancel{-6\sqrt{2}}=\boxed{2\sqrt{2}}$$

B)

$$-3\sqrt{(-2)^2}+(2\sqrt{3})^2-(4\sqrt{5})^2=$$
$$-3\sqrt{4}+2^2\sqrt{3}^2-4^2\sqrt{5}^2=$$
$$-3\cdot 2+4\cdot 3-16\cdot 5=$$
$$-6+12-80=\boxed{-74}$$

37. Упростити изразе:

А) $\frac{3}{\sqrt{7}}-\sqrt{28}$;

Б) $3\sqrt{3}+\sqrt{108}-4\sqrt{27}$.

А) $\frac{3}{\sqrt{7}}-\sqrt{28}=\frac{3}{\sqrt{7}}-\frac{\sqrt{7}}{\sqrt{7}}-\sqrt{4\cdot 7}$

$=\frac{3\sqrt{7}}{\sqrt{7}}-2\sqrt{7}$

$=\frac{3\sqrt{7}-14\sqrt{7}}{7}=-\frac{11\sqrt{7}}{7}$

В) $3\sqrt{3}+\sqrt{108}-4\sqrt{27}=$

$3\sqrt{3}+\sqrt{36\cdot 3}-4\sqrt{9\cdot 3}=$

$3\sqrt{3}+6\sqrt{3}-4\cdot 3\sqrt{3}=$

$3\sqrt{3}+6\sqrt{3}-12\sqrt{3}=-3\sqrt{3}$