

## Axial symmetry

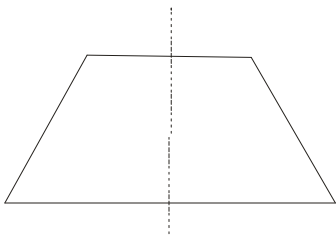
Mapping plane, in which each point  $A$  in plane mapped to point  $A'$ , symmetrical to the line  $s$ , which belongs to the plane we call **axial symmetry** with respect to the line  $s$ .

The most common mark for **axial symmetry** is  $I_s$ .

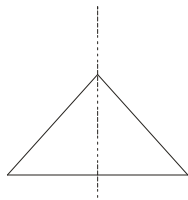
Of course, you mark this as your professor says...

For two figures  $F$  and  $F'$  in same plane we say that they are **axisymmetric** if each point  $P$  in figures  $F$  corresponds to point  $P'$  in figure  $F'$  so that  $I_s(P) = P'$ .

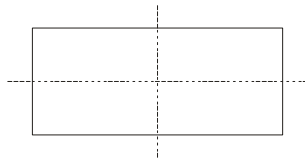
Here are some examples of axial symmetric figure with one or more axes of symmetry ...



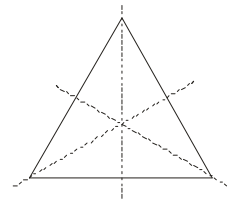
one axis of symmetry



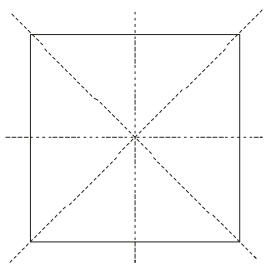
one axis of symmetry



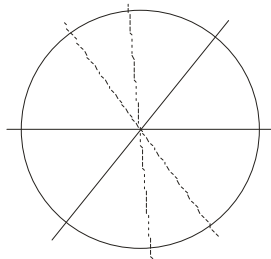
two axis of symmetry



three axis of symmetry



four axis of symmetry

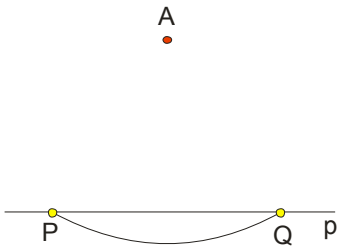


each diameter  
is axis of symmetry

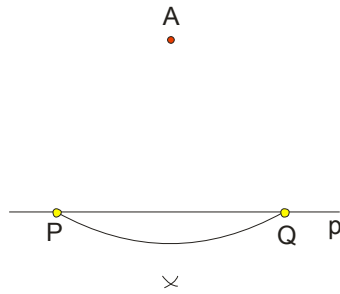
As you can see in the photos, isosceles trapezium and isosceles triangle have one axis of symmetry. A rectangle has two axes of symmetry, equilateral triangle three axes of symmetry, square has four, while each diameter of the circle is axis of symmetry.

First, let us recall a basic task:

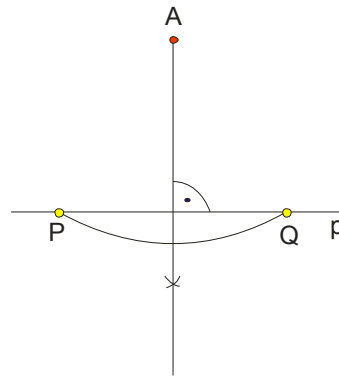
**We need to from a given point  $A$  construct a normal to a given line  $p$**



picture 1.



picture 2.



picture 3.

From point  $A$  describe the arc on the line  $p$  ( picture 1.)

Feed compass to point  $P$ , we take the hole a little larger than half the distance  $PQ$  and describe a small arc. Draw the same arc from point  $Q$  (picture 2.)

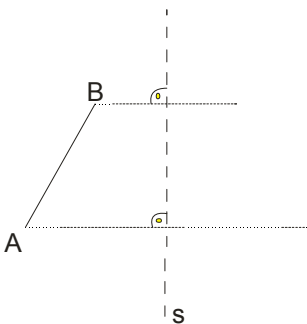
The intersection of these arcs connect with point  $A$  and that's normal ... (picture 3.)

Of course, if your teacher allows, it is easier to use a right angle to the triangle...

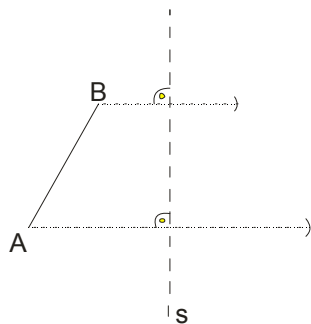
**Example 1.**

**For given along  $AB$  construct along  $A'B'$  symmetrical with respect to the right  $s$  that does not cut along.**

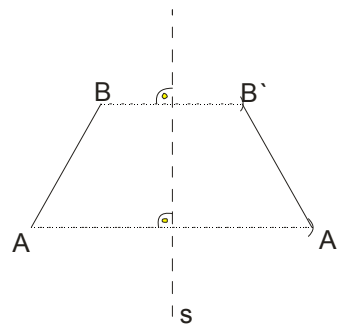
**Solution**



picture 1.



picture 2.



picture 3.

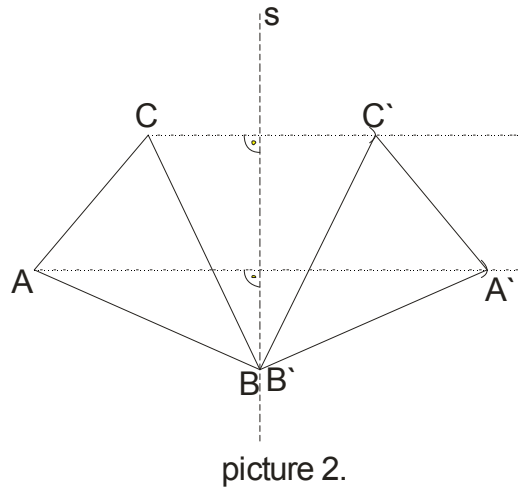
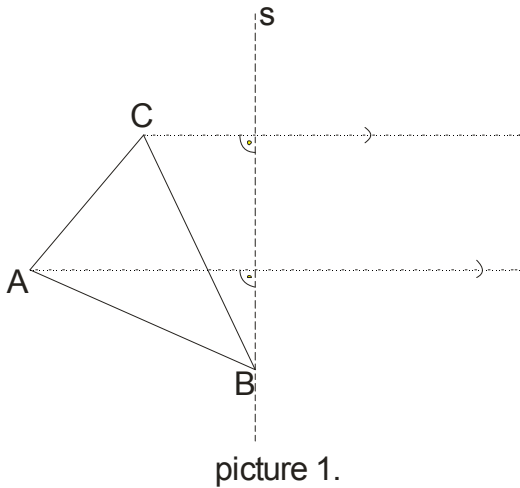
Remark

Be careful, this is a structural assignment, which means that we should do all the steps: analysis, construction, proof, discussion. We will explain how specifically to enhance the symmetry and you, again repeat, if your professor asks, must do everything in detail ...

**Example 2.**

**Given triangle ABC. Construct it symmetrical triangle to the line which passing through point B and does not cut AC.**

**Solution:**

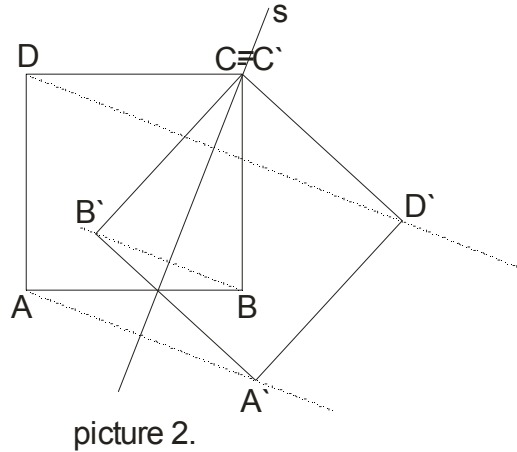
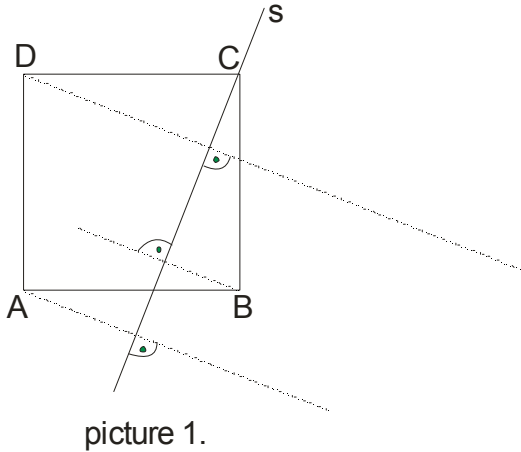


Here we have one important thing to remember: **if a point is on the axis of symmetry, then it does not have to be transfer because its axial symmetric point is the very point, that is,  $B \equiv B'$ .**

**Example 3.**

**Line  $s$ , axis of symmetry, pass through  $C$  in square  $ABCD$  and cuts  $AB$ . Construct a square symmetrical to square  $ABCD$ .**

**Solution:**

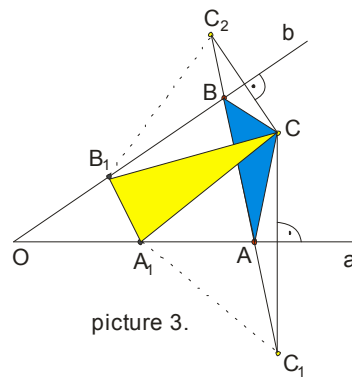
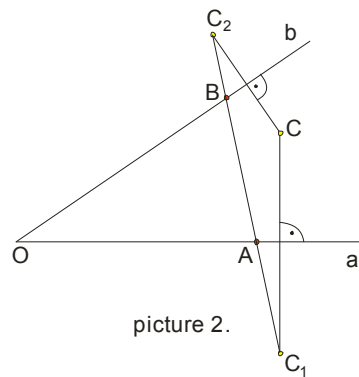
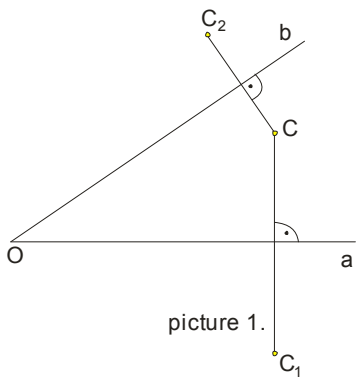


Point  $C$  is on the axis, so  $C \equiv C'$  and for the other points we know process ...

**Example 4.**

**Given the sharp angle  $OAB$  in it point  $C$ . Construct points  $A$  and  $B$ ,  $A \in a, B \in b$  so that the volume of the triangle  $ABC$  be the smallest.**

**Solution:**



First, we construct points  $C_1$  and  $C_2$  which are symmetric with point  $C$  in relation to  $Oa$  and  $Ob$ . ( picture 1.)

Merge along  $C_1 C_2$ . The intersection  $C_1 C_2$  with  $Oa$  and  $Ob$  gives us the points  $A$  and  $B$  ( picture 2.).

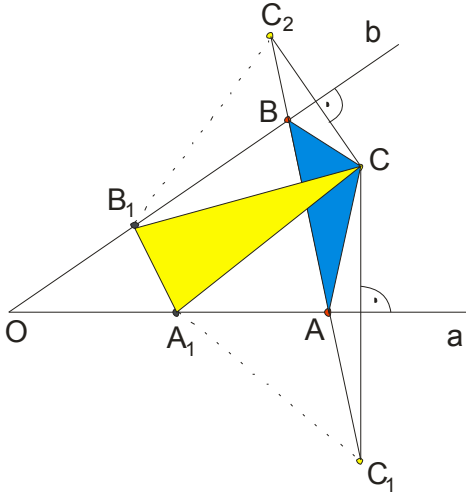
Merge points A, B and C to get the triangle of least volume ( picture 3.)

Of course, now ask ourselves **why this triangle is just with the smallest volume?**

$O = AB + AC + BC$ , as is  $AC = A C_1$  and  $BC = B C_2$ ,

$O = AB + A C_1 + B C_2$ , volume is along  $C_1 C_2$ .

If you would take some two other points  $A_1$  and  $B_1$ , we would have:



Volume of this triangle would be:  $O = A_1 B_1 + A_1 C + B_1 C$  and as:  $A_1 C = A_1 C_1$  and  $B_1 C = B_1 C_2$ , we have:

$O = A_1 B_1 + A_1 C_1 + B_1 C_2$  and that the broken line is certainly shorter than  $C_1 C_2$ . ( so picture)

**Example 5.**

The two ships, *ship A* and *ship B* are anchored to the sea, not far from the *shore p* (straight line).

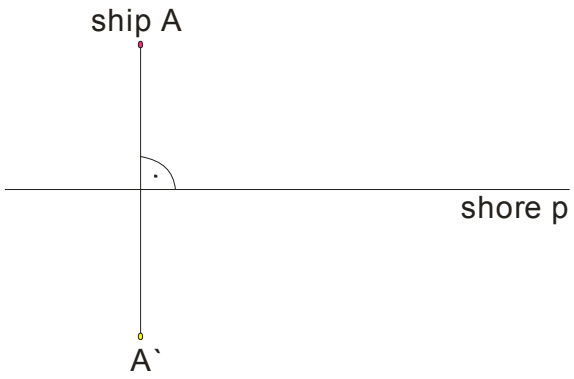
From the *boat A* canoe need to carry a passenger to shore and then come to *the ship B*.

Determine (construct) the shortest path which canoe have to sail to make a set task.

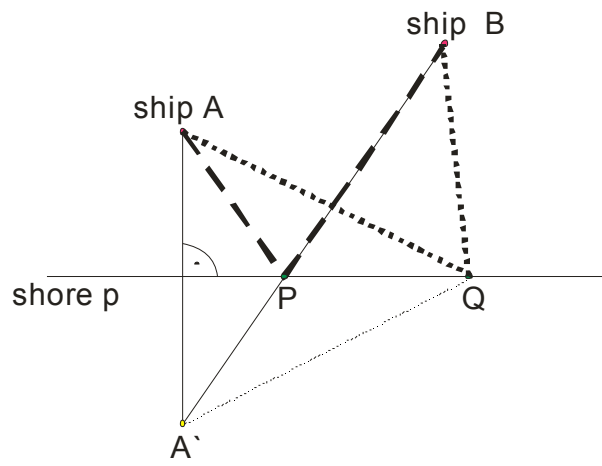
**Solution:**

Way of thinking is similar to the previous task:

ship B



picture 1.



picture 2.