Vectors in the plane

Simply put, **vectors are directed long.**

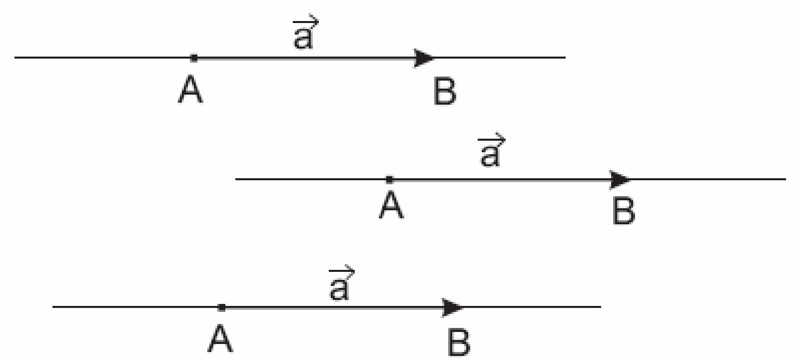
Characteristic of vectors are :

- **direction**
- **intensity**
- **the beginning and the end of vectors**

A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an **initial point** *A* with a **terminal point** *B*

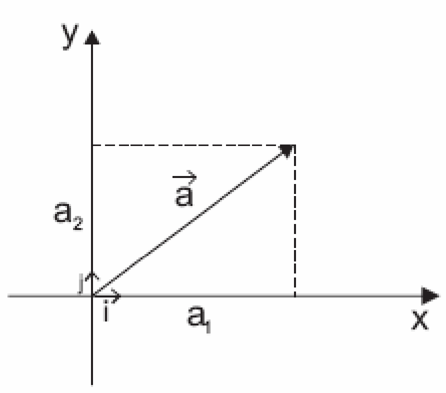
A vector **intensity** is its length and is usually marked with $|\vec{a}|$

A vector is what is needed **to "carry"** the point *A* to the point *B*; the Latin word *vector* means **"one who carries"**.



Marks $\overrightarrow{AB} = \vec{a}$

How to specify a vector?



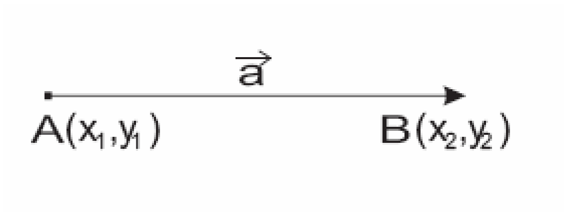
$\vec{a} = a_1\,\vec{i} + a_2\,\vec{j}$ or simply $\vec{a} = (a_1, a_2)$; intensity is $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$

$\vec{i}$ i $\vec{j}$ are unit vectors (Oort) used for the expression of other vectors.

$\vec{i}$=(1,0) and intensity of this vector is $|\vec{i}|$=1

$\vec{j}$=(0,1) and also $|\vec{j}|$=1

How to express the vector if they are given the coordinates of its beginning and end? (an **initial point** *A* with a **terminal point** *B)*



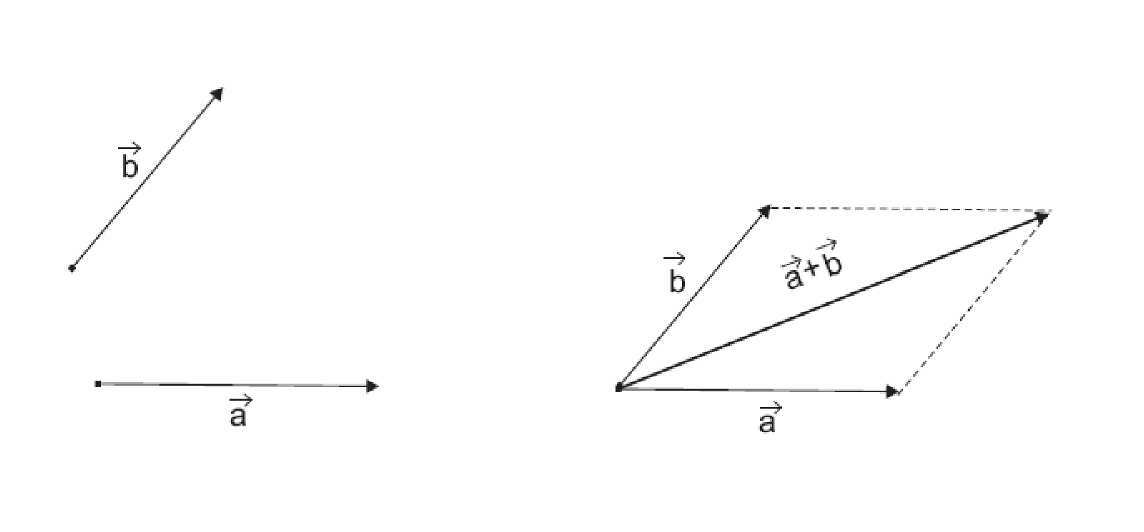
$\vec{a}$=(x_2-x_1, y_2-y_1) and its intensity is $|\vec{a}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Addition and subtraction of vectors

For addition and subtraction of vectors we have two rules:

1) Parallelogram rule

By parallel moving bring two vectors to together beginning. Of them, as the pages set up a parallelogram. Diagonal of the parallelogram is the sum (it is diagonal, which starts from the composition of the two vectors).

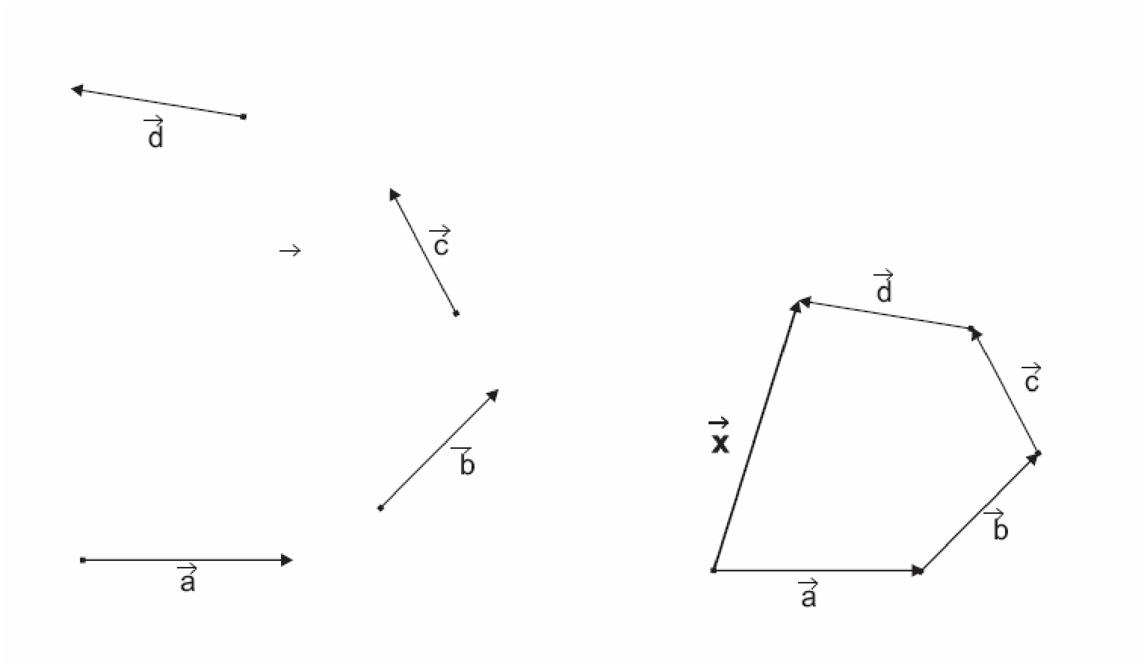


2) Polygon Rule (concatenation)

At the end of the first vector parallel moving bring the beginning of the second, the end of the second bring the beginning of the third vector......

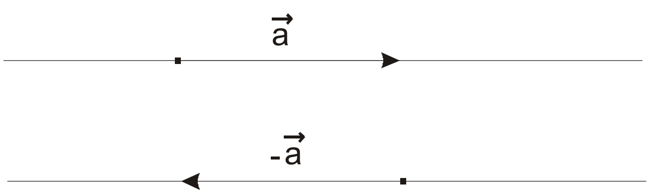
Resultant (their sum) **is a vector that connects the beginning of the first and the end of last vector**.

Here's to the picture:



Our proposal is to use concatenation rule, because it is easier to estimate...

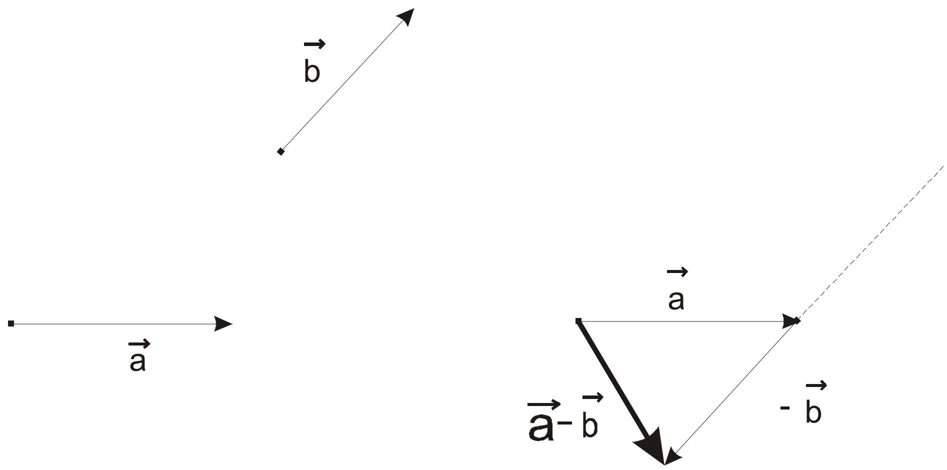
Each vector has its **opposite vector**, who has the same direction and intensity, but opposite direction to the initial vector.



$-\vec{a}+\vec{a}=0$ and $\vec{a}+(-\vec{a})=0$

How to subtract two vectors?

Suppose you are given vectors $\vec{a}$ i $\vec{b}$,. The procedure is similar to the addition of vectors (concatenation rule) only instead of the vector $+\vec{b}$ to the end of the first we put - $\vec{b}$.

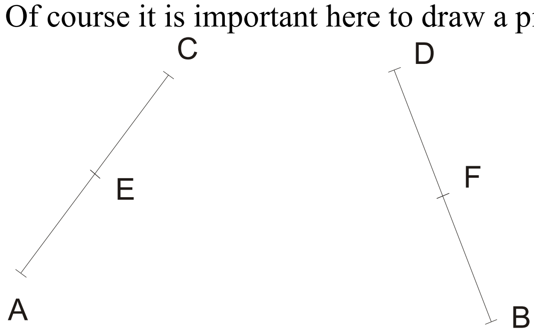


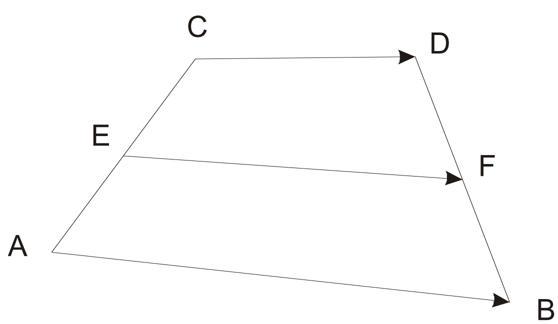
Example:

1) AC and BD are given long . The points E and F are in the middle of these two long. Prove that :
$\overrightarrow{AB}+\overrightarrow{CD}=2\overrightarrow{EF}$

Solution:

Of course it is important here to draw a picture to do the task!





The idea is to expres vector $\overrightarrow{EF}$ on both sides , and to gather them.

$$\left.\begin{aligned}\overrightarrow{EF}&=\overrightarrow{EA}+\overrightarrow{AB}+\overrightarrow{BF}\\ \overrightarrow{EF}&=\overrightarrow{EC}+\overrightarrow{CD}+\overrightarrow{DF}\end{aligned}\right\}\ +$$

$2\overrightarrow{EF}=\overrightarrow{AB}+\overrightarrow{CD}$ vectors EA i EC are the opposite , and vectors BF and DF also. They give a zero vector.

Computing addition and subtraction of vectors go easy:

If $\vec{a} = a_1\vec{i} + a_2\vec{j}$ or $\vec{a} = (a_1, a_2)$ and $\vec{b} = b_1\vec{i} + b_2\vec{j}$, or $\vec{b} = (b_1, b_2)$

$\vec{a} + \vec{b} = (a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$
$\vec{a} - \vec{b} = (a_1, a_2) - (b_1, b_2) = (a_1 - b_1, a_2 - b_2)$

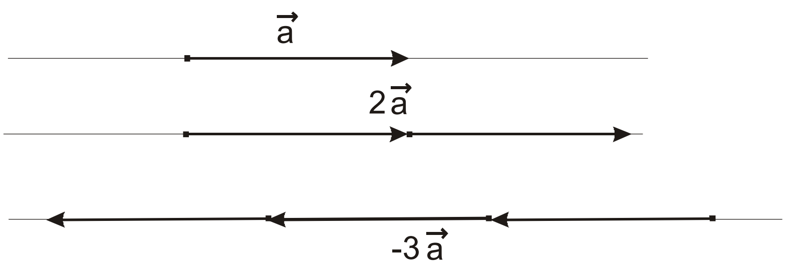
Scalar multiplication of vectors

Product scalar k and vector $\vec{a}$ is a vector k$\vec{a}$ (or $\vec{a}$k) which has the same direction as the vector $\vec{a}$, intensity $|k\vec{a}| = |k||\vec{a}|$ and direction:

- same as the vector $\vec{a}$ if k>0
- the opposite of the vector $\vec{a}$ if k<0

Example: We have vector $\vec{a}$, Find : 2$\vec{a}$ and -3$\vec{a}$

Solution:



Each vector $\vec{a}$ can be expressed as $\vec{a} = |\vec{a}|\overrightarrow{a_0}$, where is $\overrightarrow{a_0}$ unit vector of vector $\vec{a}$.

Linear dependence of vectors

If $k_1, k_2, \ldots, k_n$ real numbers and $\overrightarrow{x_1}, \overrightarrow{x_2}, \ldots, \overrightarrow{x_n}$ vectors different from zero, then the sum of:

$$k_1\overrightarrow{x_1} + k_2\overrightarrow{x_2} + \ldots + k_n\overrightarrow{x_n}$$

called a **linear combination** of vectors $\overrightarrow{x_1}, \overrightarrow{x_2}, \ldots, \overrightarrow{x_n}$

Equate this linear combination with zero:

$$k_1\overrightarrow{x_1} + k_2\overrightarrow{x_2} + \ldots + k_n\overrightarrow{x_n} = 0$$

i) If $k_1 = k_2 = \ldots = k_n$, then the vectors $\overrightarrow{x_1}, \overrightarrow{x_2}, \ldots, \overrightarrow{x_n}$ **linearly independent**

ii) If at least one of $k_1, k_2, \ldots, k_n$ different from zero then the vectors $\overrightarrow{x_1}, \overrightarrow{x_2}, \ldots, \overrightarrow{x_n}$ **linearly dependent**

Valid**:**

Two vectors are **collinear** if and only if they are linearly dependent (collinear means that lie on the same real).

Vectors $\vec{x}$, $\vec{y}$, $\vec{z}$ lie in the same plane if and only if they are linearly dependent .

Decomposition of the vector on components

If the vectors $\vec{x}$ and $\vec{y}$ are linearly independent vectors of a plane, then for each vector $\vec{z}$ in the plane, there are unique numbers p and q such that:

$$\vec{z} = p\vec{x} + q\vec{y}$$

Example:

Vector $\vec{v}$**=(4,2)** decompose to the vectors $\vec{a}$**=(2,-1) and** $\vec{b}$**= (-4,3)**

Solution:

$\vec{v} = p\vec{a} + q\vec{b}$

(4,2) = p(2,-1) + q(-4,3)
(4,2) = (2p,-p) + (-4q,3q)

Here we make the system:

4=2p – 4q
2=-p + 3q

2p –4q = 4
-p + 3q = 2

p – 2q = 2
-p+3q = 2

q = 4

2p-4q = 4 , pa je 2p – 16 = 4 , pa 2p = 20 and finally p = 10 .

Therefore, the decomposition of the vector is $\vec{v} = 10\vec{a} + 4\vec{b}$

If you have studied this topic, see the immediately following, in which tasks are solved...