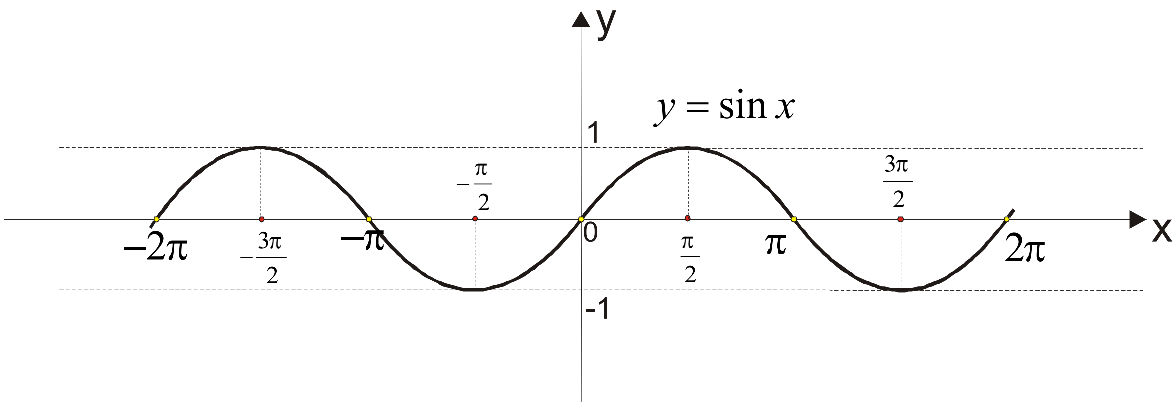
Trigonometric functions-graphics (part I)

$$\boxed{y = a\sin(bx+c)}$$

Recall the basic graphics $y = sinx$ and its properties.



Features**:**

- Function defined for all x, that is : $x \in (-\infty, \infty)$

- Set the value function is an interval $[-1,1]$, that is, a function is limited $-1 \le \sin x \le 1$

- Sin x is a periodic function with the main period 2π

- **Zero function** (where the graph cuts x axis) are $x = 0, x = \pm\pi, x = \pm 2\pi...$ or it can be written, taking into account the periodicity $x = k\pi$ $(k = 0, \pm1, \pm2, ...)$

- **Maximum value** functions are $-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, ...$ we can write: $x = \frac{\pi}{2} + 2k\pi$ $k \in Z$

- **The minimum value** function has in $-\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, ...$ we can write: $x = -\frac{\pi}{2} + 2k\pi$ $k \in Z$

- *sinx* **increases** in intervals $[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi]$, $k \in Z$

- *sinx* **decreases** in the intervals $[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi]$, $k \in Z$

- Function is **positive**, *sinx > 0* for $x \in (2k\pi, (2k+1)\pi)$ $k \in Z$

- Function is **negative** , *sinx < 0* for $x \in ((2k-1)\pi, 2k\pi)$ $k \in Z$

- The graph is called a **sinusoid**

Trigonometric function $y = a\sin(bx + c)$ **we will learn to draw in two ways.**

The first method consists in the fact that we start from the initial graphics *y = sinx* and depending on the numbers *a, b* and *c* we moving graphics (we will learn how) and the *second way* is to directly test points (zero function, max, min ...) but for it we need to knowto solve trigonometric equations.

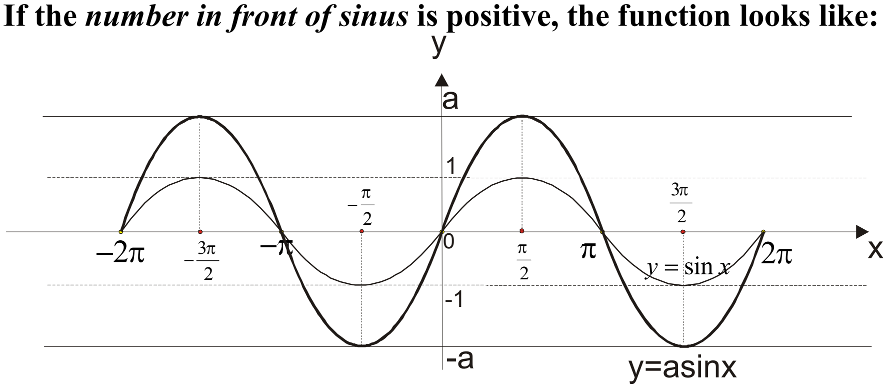
First, note and write down the numbers *a, b* and *c.*

$\boxed{y = a\sin x}$

Number ***a,*** *which* is in front of the sinuses is called ***the amplitude*** and it is the maximum distance point graphics from x-axis.

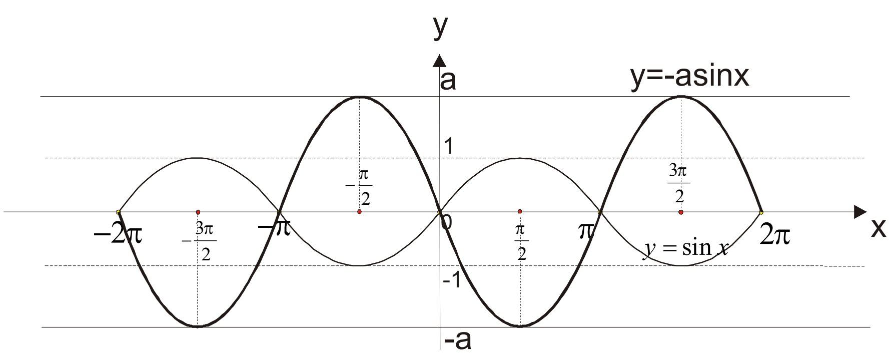
For the function *y = sinx* is the number *a = 1* and we see that the graphic is just limited by -1 and 1

If the *number in front of sinus* is positive, the function looks like:



Therefore, the zero remains in place, while the max and min "extend" to the *point a and –a.*

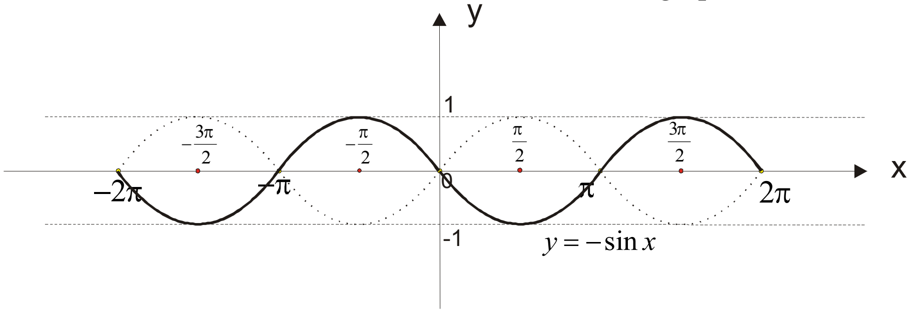
If the *number in front of sinus* is negative, the function looks like:



Watch out, here is the graph turns!

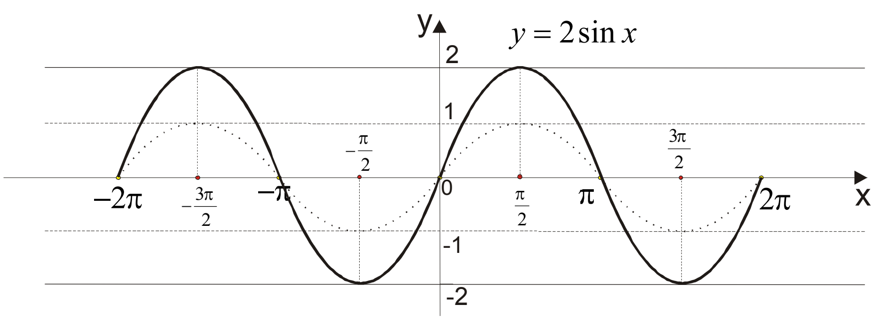
Example 1. **Draw a graph** $y = -\sin x$

Here we have ***a = -1.*** This indicates that the initial graph is turns.



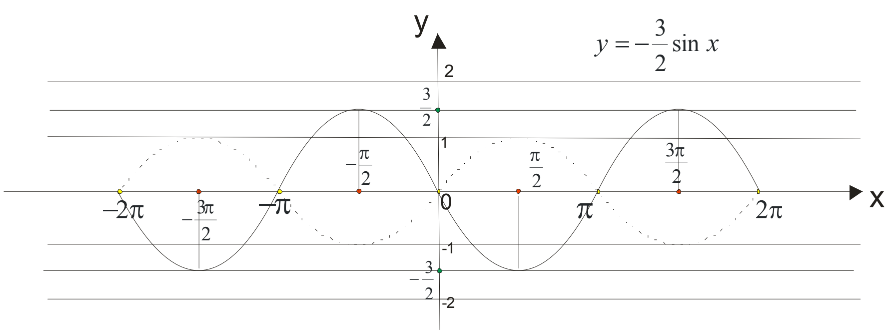
Example 2. **Draw a graph** $y = 2\sin x$

Now is ***a = 2***. This means that the function of the y-axis goes from -2 to 2 and that the graph does not turn.



Example 3. **Draw a graph** $y = -\frac{3}{2}\sin x$

We have that $a = -\frac{3}{2}$.



$\boxed{y = \sin bx}$

Periodicity functions $y = a\sin(bx + c)$ directly follows from the periodicity of *y=sinx.*

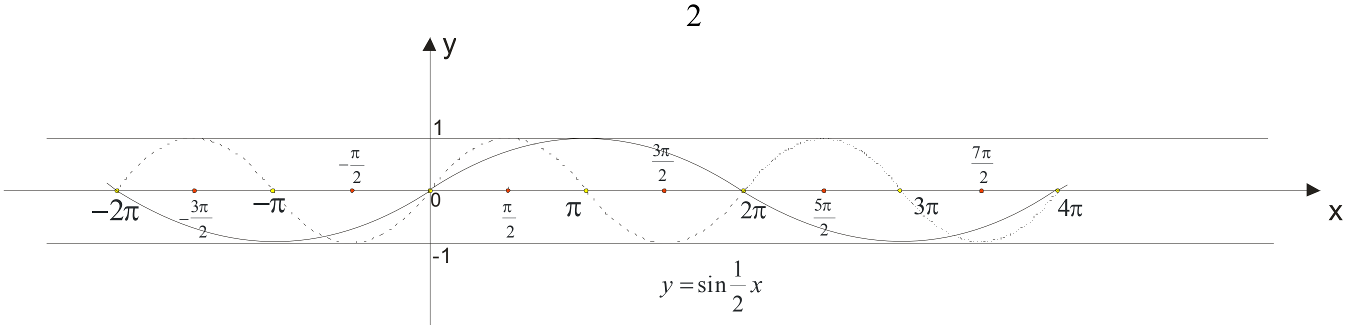
The main period of $y = a\sin(bx + c)$ **is calculated by the formula** $\boxed{T = \frac{2\pi}{b}}$.

The number b is called the *frequency,* and shows how the whole wave is on the interval $[0, 2\pi]$

Hence, our referred is to observe the number *b,* and insert it into $T = \frac{2\pi}{b}$ to get the base period.

<u>Example 4.</u> **Draw a graph** $y = \sin\frac{1}{2}x$

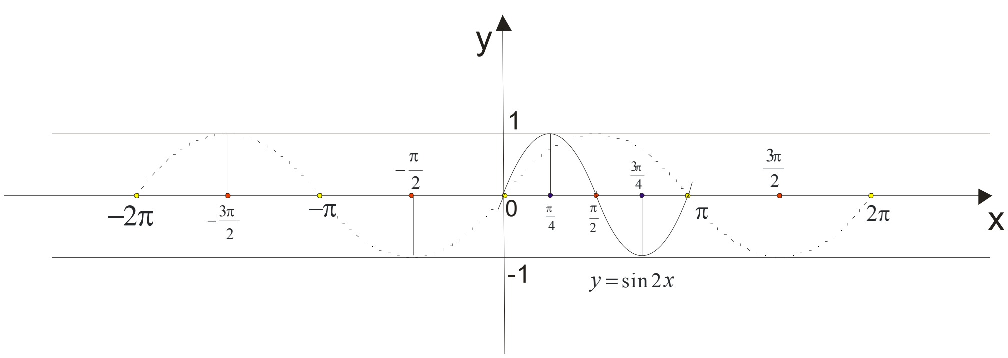
We notice that here $b = \frac{1}{2}$. Then $T = \frac{2\pi}{b} \to T = \frac{2\pi}{\frac{1}{2}} \to \boxed{T = 4\pi}$



Initial function *y = sinx* is here given intermittent. What happened to her? She "stretched" because $T = 4\pi$.

<u>Example 5.</u> **Draw a graph** $y = \sin 2x$

As is $b = 2$ then it the main period will be $T = \frac{2\pi}{b} \to T = \frac{\cancel{2}\pi}{\cancel{2}} \to \boxed{T = \pi}$



$\boxed{y=\sin(x+c)}$ or $\boxed{y=\sin(bx+c)}$ **(Number c is called the *initial stage)***

Again, of course, first read from the given function values for b and c. Then determine the value for $\frac{c}{b}$.

$y=\sin(bx+c)$ **is obtained by moving graphics** $y=\sin bx$ **along the x axis and (watch this):**

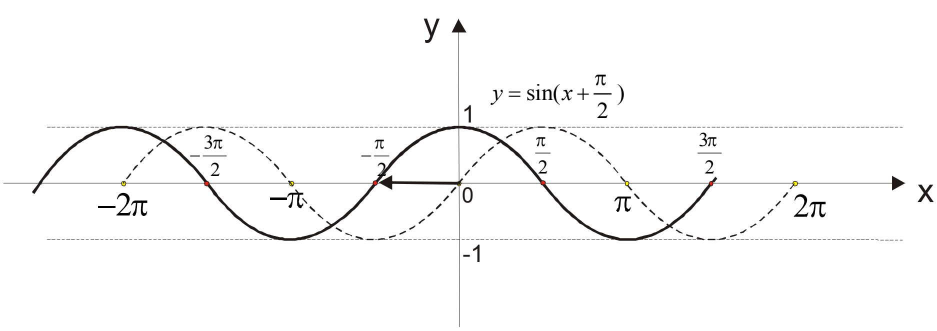
i) in a positive direction (right) if the value $\frac{c}{b}$ is negative

ii) in a negative direction (left) if the value of $\frac{c}{b}$ is positive

__**Example 6.**__ **Draw a graph** $y=\sin(x+\frac{\pi}{2})$

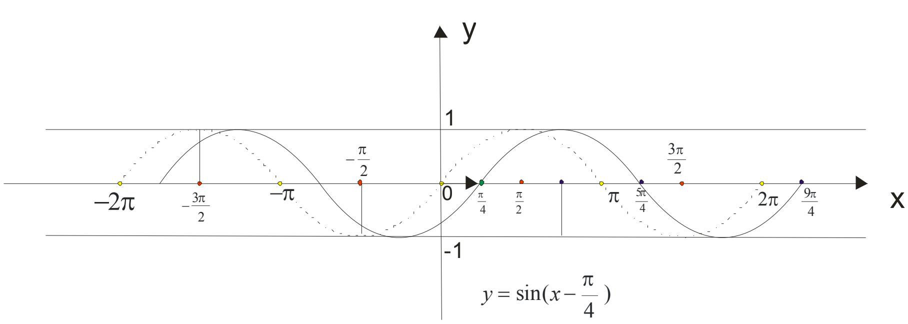
Here are $a=1,\ b=1,\ c=\frac{\pi}{2}$. Value of expression $\frac{c}{b}$ is $\frac{c}{b}=\frac{\frac{\pi}{2}}{1}=\frac{\pi}{2}$. **What does this mean?**

Since the value of this expression is positive, the initial graph of *y = sinx* move for $\frac{\pi}{2}$ in left.



__**Example 7.**__ **Draw a graph** $y=\sin(x-\frac{\pi}{4})$

$a=1, b=1, c=-\frac{\pi}{4}\rightarrow\boxed{\frac{c}{b}=-\frac{\pi}{4}}$ Move ***y = sinx*** **for** $\frac{\pi}{4}$ right .

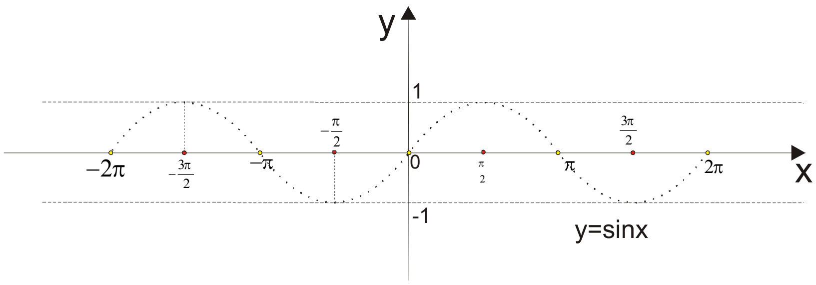


Example 8. **Draw a graph** $y = \sin(2x - \frac{\pi}{2})$

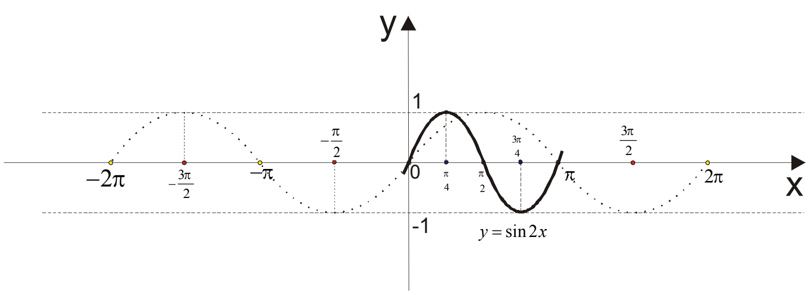
Here are $a=1$, $b=2$ and $c = -\frac{\pi}{2}$

$$T = \frac{2\pi}{b} \rightarrow T = \frac{\cancel{2}\pi}{\cancel{2}} \rightarrow \boxed{T = \pi} \quad \text{and} \quad \frac{c}{b} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$$

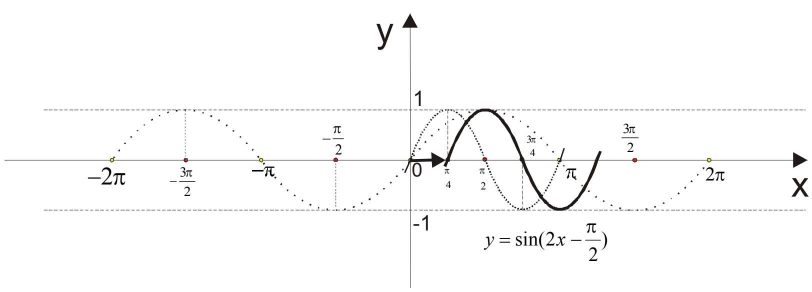
We have to draw **three** graphics : *y=sinx* (**slika 1**.) then *y=sin2x* (**slika 2**.) and $y = \sin(2x - \frac{\pi}{2})$ (**slika 3**.)



slika 1.



slika 2.



slika 3.

On picture 2. we see that the graph of sine function "assembled" for period $T = \pi$.

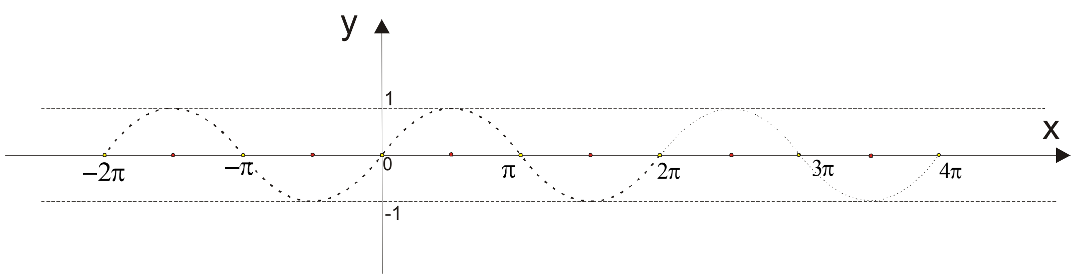
On picture 3. has been made moving graphic *y=sin2x* for $\frac{\pi}{4}$ in right .

Your professor will probably ask you to apply all three graphics on one picture ... We intentionally draw three pictures for better understand…

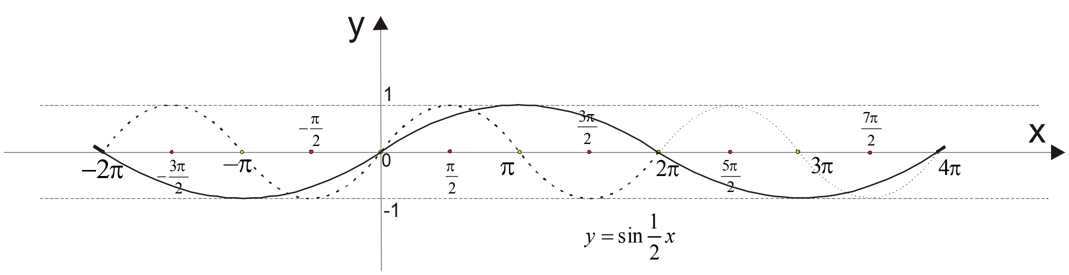
Example 9. **Draw a graph** $y=\sin(\frac{1}{2}x+\frac{\pi}{4})$

$$a=1, b=\frac{1}{2}, c=\frac{\pi}{4}$$

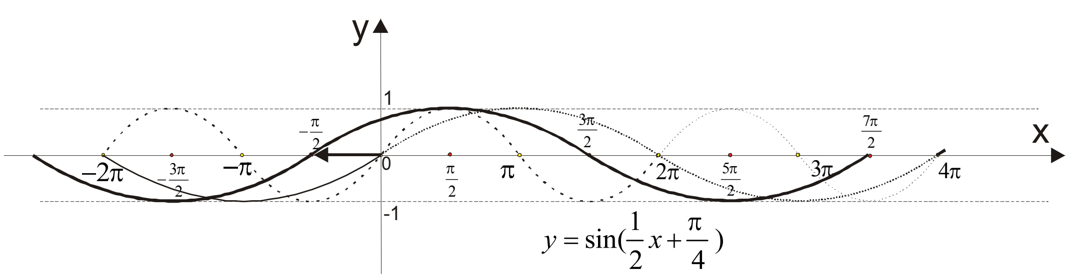
$$T=\frac{2\pi}{b}\rightarrow T=\frac{2\pi}{\frac{1}{2}}\rightarrow \boxed{T=4\pi} \quad \text{and} \quad \frac{c}{b}=\frac{\frac{\pi}{4}}{\frac{1}{2}}=\frac{2\pi}{4}=\frac{\pi}{2}$$



slika 1.



slika 2.



slika 3.

Now we have the knowledge to draw the whole graph $y=a\sin(bx+c)$. See next file:

Trigonometric functions-graphics (part II)