Trigonometrical circle

Angles can be measured in degrees and radians. With the idea of the degree we learn more in elementary school and if you remember, we have divided it in minutes and seconds (1^0=60\` , 1\`=60\`\`). To explain what is radian, We will observe circle with radius R . We have formula for volume: O= 2Rπ, and know that $\pi \approx 3,14$.

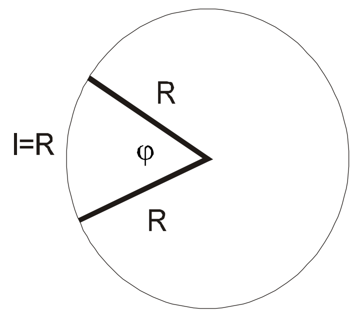
If you take part of the circles (circular arc), which is just the length of R, it corresponds to a central angle φ.

Measure the angle of the central port that corresponds to the length of R is one radian.

It is clear that the full angle has 2π radians.Or:

$360^0=2\pi$ radians

$180^0=\pi$ **remember**



$$1^0 = \frac{\pi}{180} radians$$

So :
$$1` = \frac{\pi}{180*60} radians$$

$$1`` = \frac{\pi}{180*60*60} radians$$

And vice versa: $1 rad = \frac{180^0}{\pi} \approx 57^0 17`45``$

Example 1: Find measure of angle in radians:

$a) 75^0$
$b) 245^0$
$c) 82^0 30`$

Solution:

a) How is $1^0 = \frac{\pi}{180} radians$ it is $75^0 = 75\frac{\pi}{180} = \frac{5\pi}{12}$

b) $245^0 = 245\frac{\pi}{180} = \frac{49\pi}{36}$

c) $82^0 30` = 82\frac{\pi}{180} + 30\frac{\pi}{180*60} = \frac{11\pi}{24}$

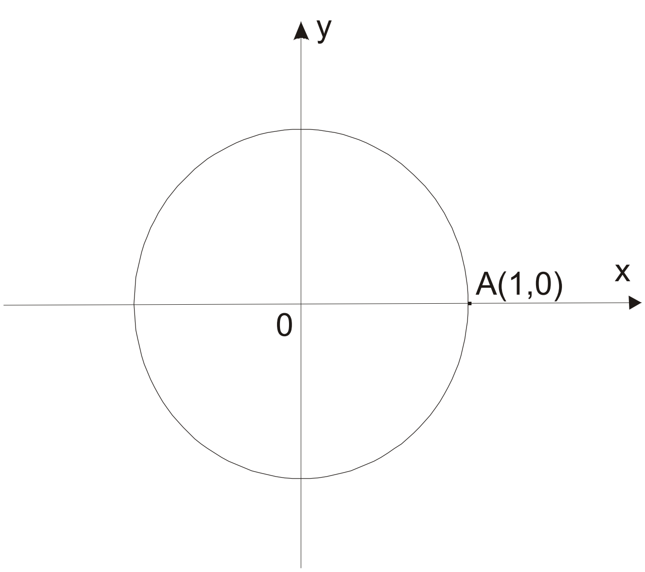
Example 2: Find the measure in degrees:

$$a)\frac{3\pi}{4}$$
$$b)\frac{11\pi}{6}$$
$$c)5radians$$

Solution:

$$a)\frac{3\pi}{4}=\frac{3*180}{4}=135^0$$
$$b)\frac{11\pi}{6}=\frac{11*180}{6}=330^0$$
$$c)5radians=5(57^0 17`45``)$$
$$=285^0 85`225``$$
$$=285^0 88`45``$$
$$=286^0 28`45``$$

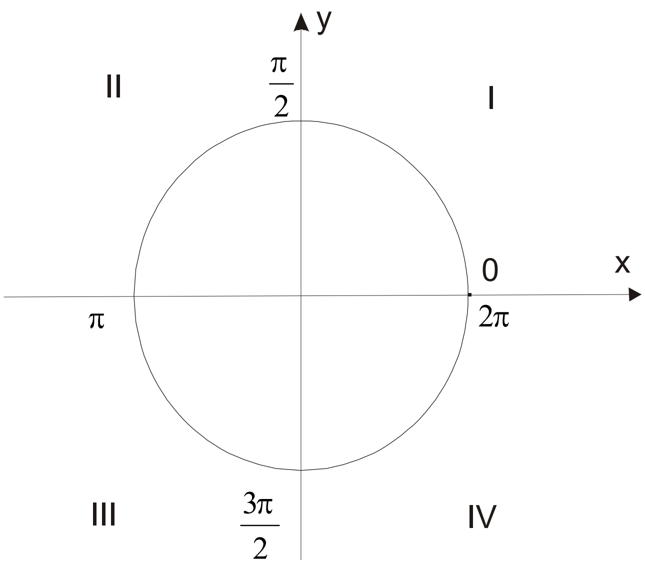
Trigonometrical circle is a circle of radius 1 which center is in piont (0,0).



Point A(1.0), which belongs to the trigonometric circle is called a starting point. On trigonometric circle we are going to observe different bows, which begin in point A.

If you go clockwise- it is positive direction.

If you go in the opposite direction from the direction of movement clockwise- it is negative direction.



Angles from I : $0 < \alpha < \frac{\pi}{2}$

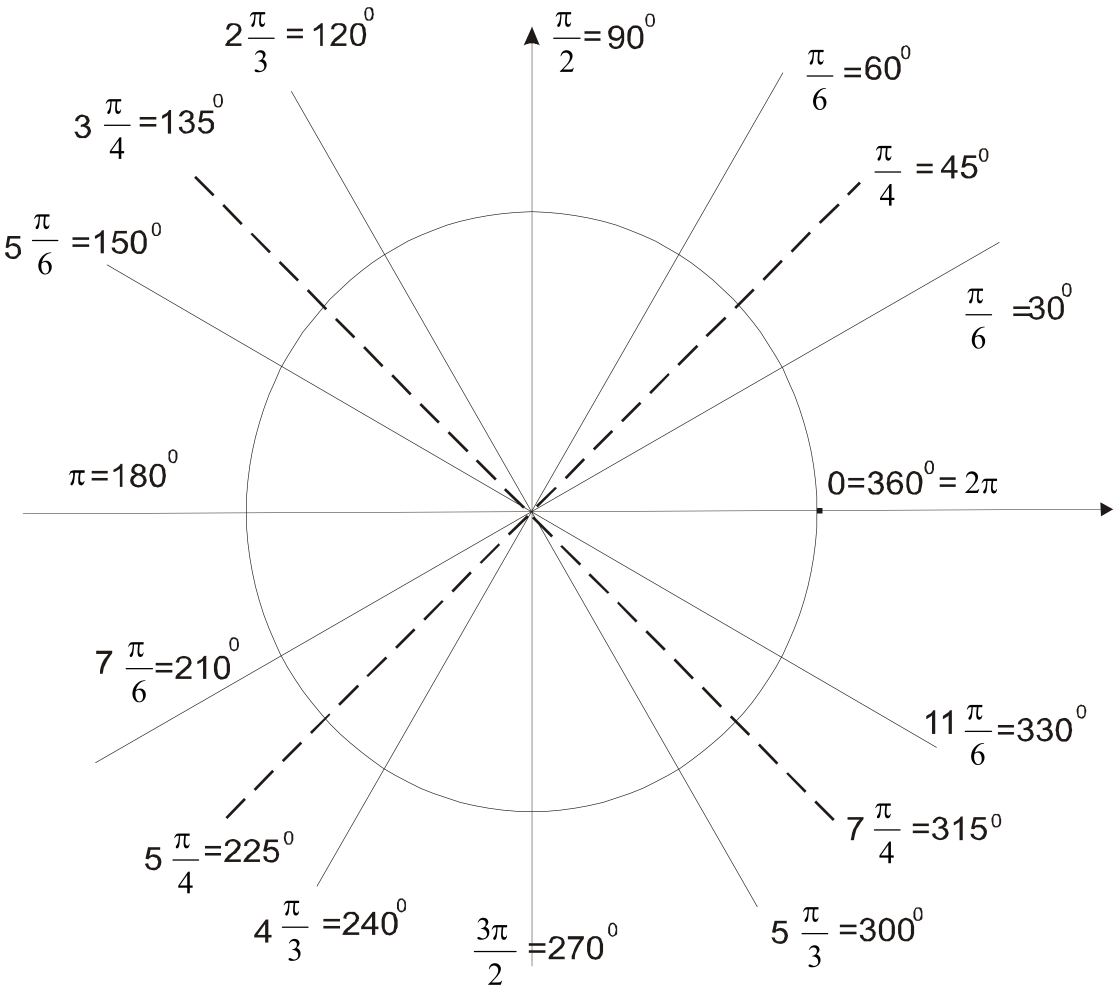
from II : $\frac{\pi}{2} < \alpha < \pi$

from III : $\pi < \alpha < \frac{3\pi}{2}$

from IV : $\frac{3\pi}{2} < \alpha < 2\pi$

Angles 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, are borders angles.

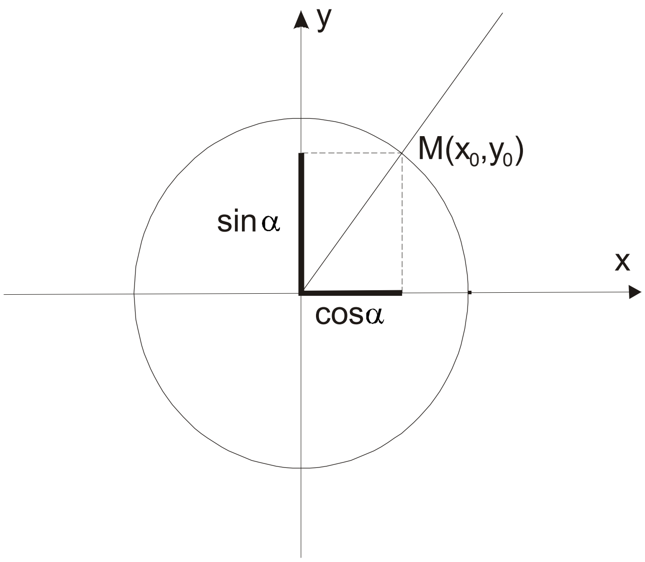
Angles which value we will find on **trigonometrical circle** are as follows:



Cosine and sine of arbitrary angle

For any arbitrary angle is always one shank on x, ie, going throught A (1.0), second shank is cutting trigonometric circle in a point $M(x_0,y_0)$. From that point draw normal on x and y lines. That length are:

- **On the x-line** $\cos\alpha$ **(** $\cos\alpha = x_0$**)**
- **On the y-line** $\sin\alpha$ **(**$\sin\alpha = y_0$**)**



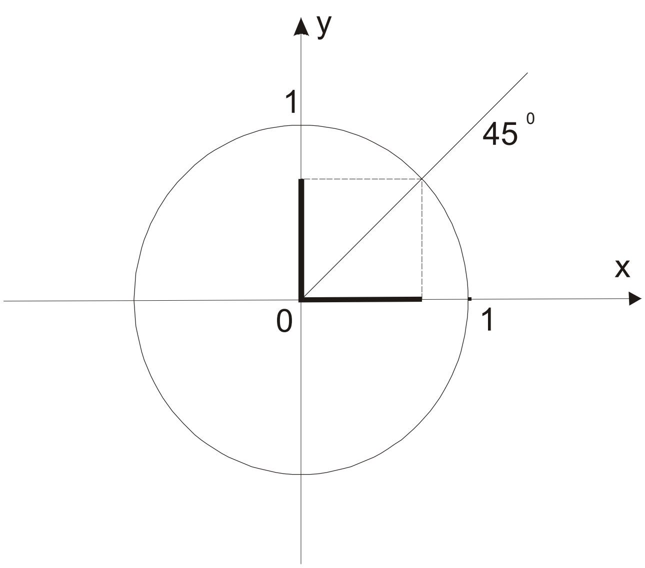
Here's our suggestions how to remember the values and to "read" them on a circle.

Remember numbers: $\frac{1}{2}$, $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{2}$ **which are ordered from the smallest to largest.**

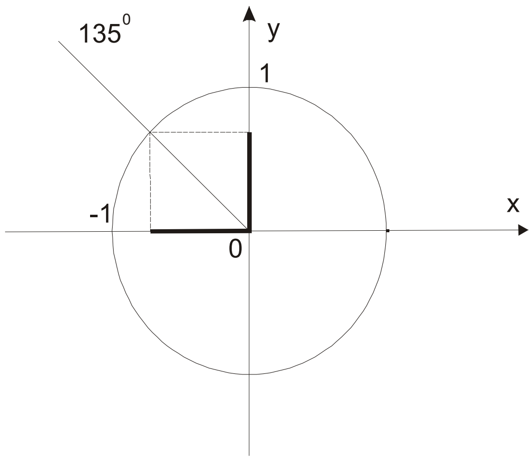
Number in the middle corresponds to the angles that are in the middle of quarters.

So, sinus and cosine angles of 45, 135, 225 and 315 degrees have value $\frac{\sqrt{2}}{2}$,only take care that the value can be $+\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$.

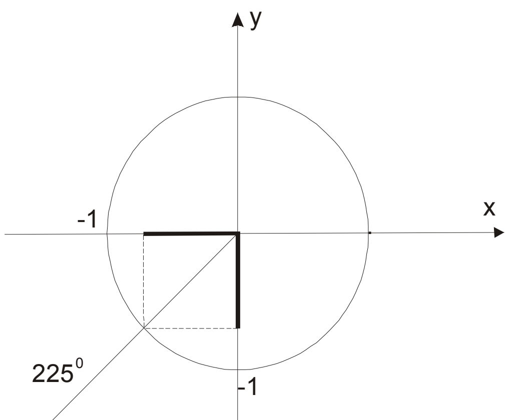
Here in the images, and will be clearer:



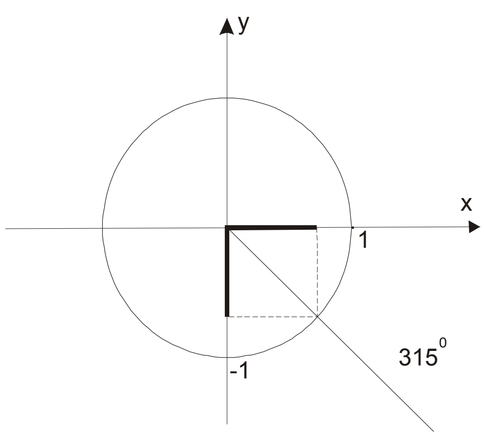
sin 45^0=cos45^0=$\frac{\sqrt{2}}{2}$



sin 135⁰$=\frac{\sqrt{2}}{2}$ a **cos 135⁰**$= -\frac{\sqrt{2}}{2}$



sin 225⁰$= -\frac{\sqrt{2}}{2}$ cos 225⁰$= -\frac{\sqrt{2}}{2}$

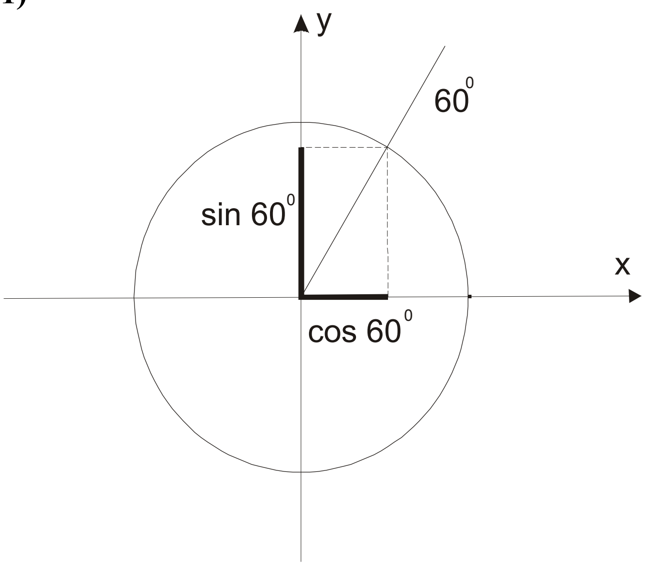


sin 315⁰$= -\frac{\sqrt{2}}{2}$ a cos 315⁰$=\frac{\sqrt{2}}{2}$

For other angles will be : $\frac{1}{2}$ or $\frac{\sqrt{3}}{2}$,of course, again to see if the + or -.

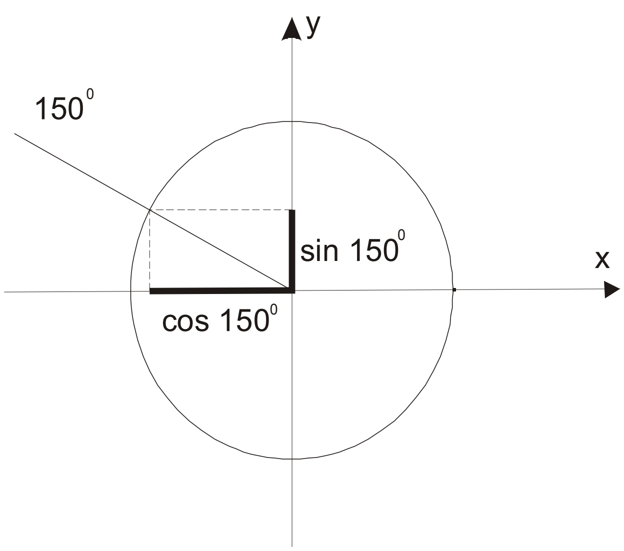
Here are a few examples:

1)



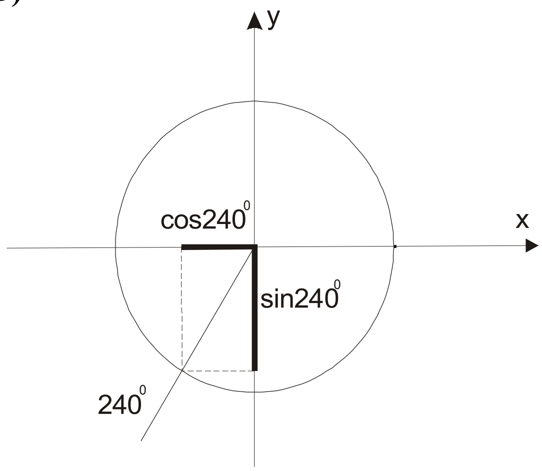
$\sin 60^0 = \frac{\sqrt{3}}{2}$ and $\cos 60^0 = \frac{1}{2}$

2)



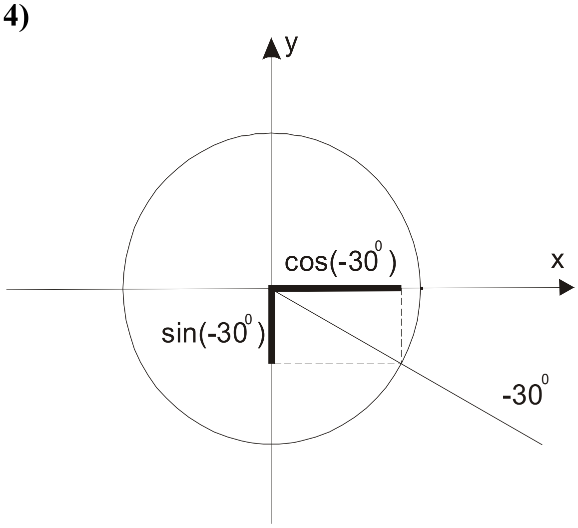
$\sin 150^0 = \frac{1}{2}$ and $\cos 150^0 = -\frac{\sqrt{3}}{2}$

3)



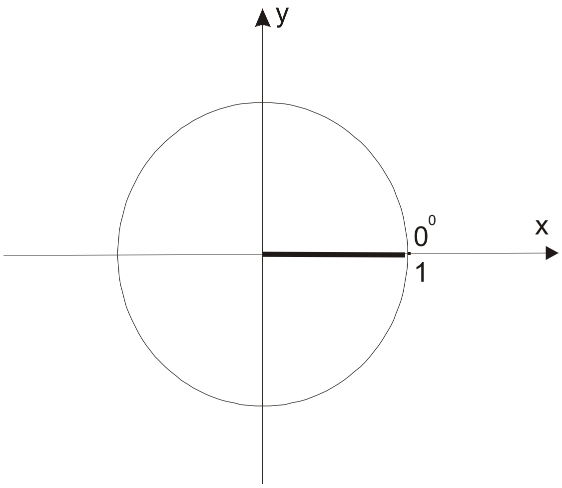
$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ and $\cos\frac{4\pi}{3} = -\frac{1}{2}$ because $\frac{4\pi}{3} = 240^0$

4)

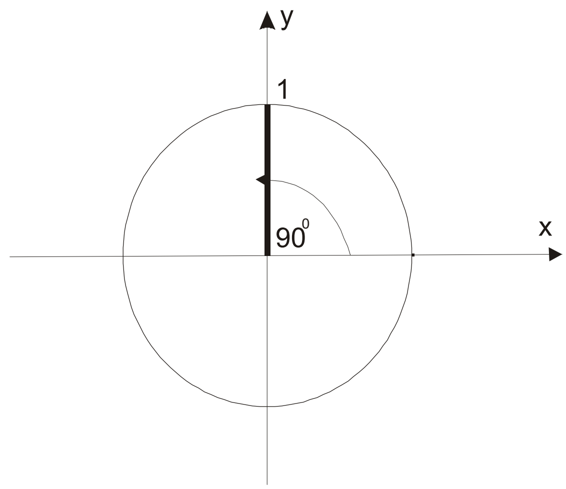


$\sin(-30^0) = \sin 330^0 = -\frac{1}{2}$ and $\cos(-30^0) = \cos 330^0 = \frac{\sqrt{3}}{2}$

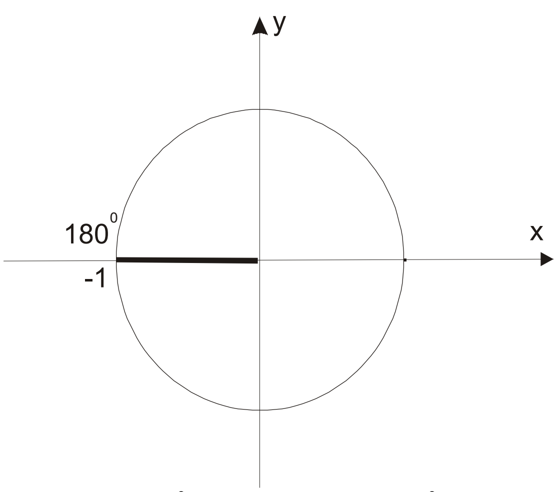
Now to see what we wiil do with angles **0,** $\frac{\pi}{2}$**,** π**,** $\frac{3\pi}{2}$:



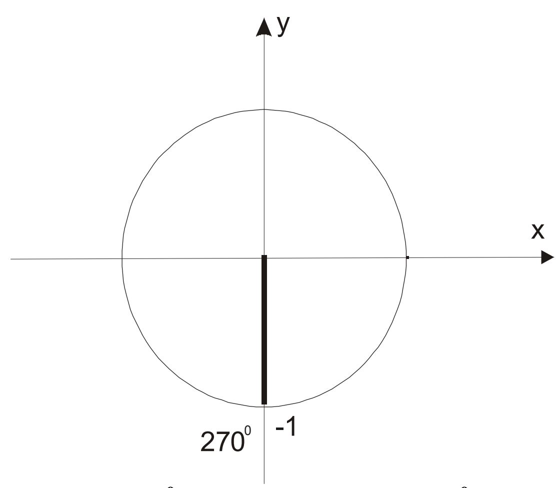
$\cos 0^0=1$ and **$\sin 0^0=0$**



$\sin 90^0=1$ and **$\cos 90^0=0$**



$\sin 180^0=0$ and $\cos 180^0= - 1$



$\sin 270^0=-1$ and $\cos 270^0=0$

Tg and ctg of arbitrary angle

We previously met with formulas: $tg\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ and $ctg\alpha = \dfrac{\cos\alpha}{\sin\alpha}$,

We concluded that $tg\alpha$ is defined for $\cos\alpha \neq 0$, ie $\alpha \neq \dfrac{\pi}{2}+\mathbf{k}\,\pi$, $\mathbf{k}\in\mathbf{Z}$;

and $ctg\alpha$ for $\sin\alpha \neq 0$, ie $\alpha \neq \mathbf{k}\,\pi$, $\mathbf{k}\in\mathbf{Z}$

This means that if you know to find $\sin\alpha$ and $\cos\alpha$, then you know to find $tg\alpha$ and $ctg\alpha$

Example 1.

Find:

a) $\operatorname{tg}\frac{\pi}{4}$

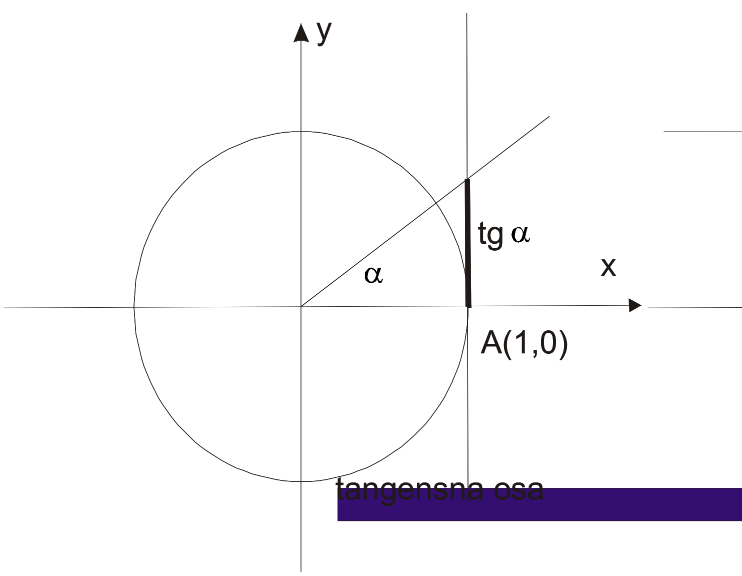
b) $\operatorname{ctg} 300^0$

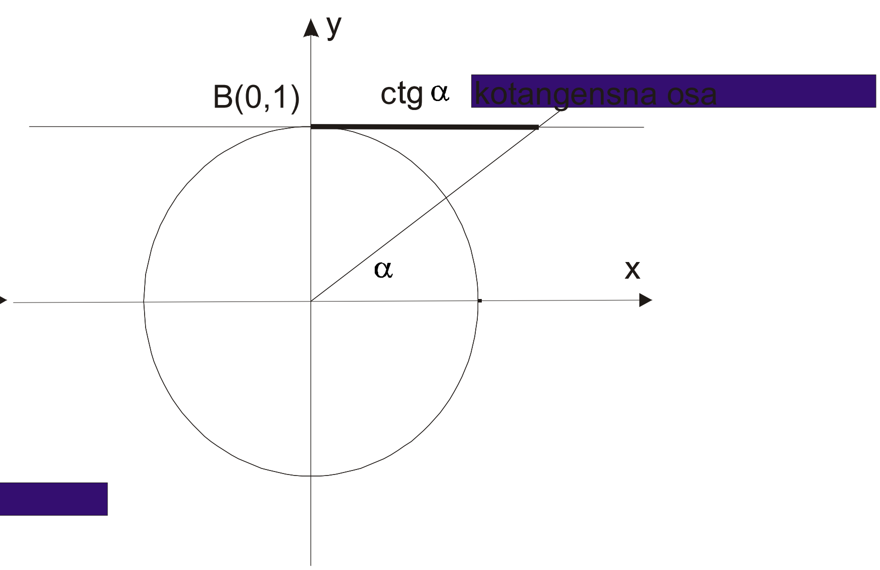
Solution:

a) $\operatorname{tg}\frac{\pi}{4} = \operatorname{tg} 45^0 = \frac{\sin 45^0}{\cos 45^0} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

b) $\operatorname{ctg} 300^0 = \frac{\cos 300^0}{\sin 300^0} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

We will learn now where to "read" values for tangent and cotangent on trigonometric circle.



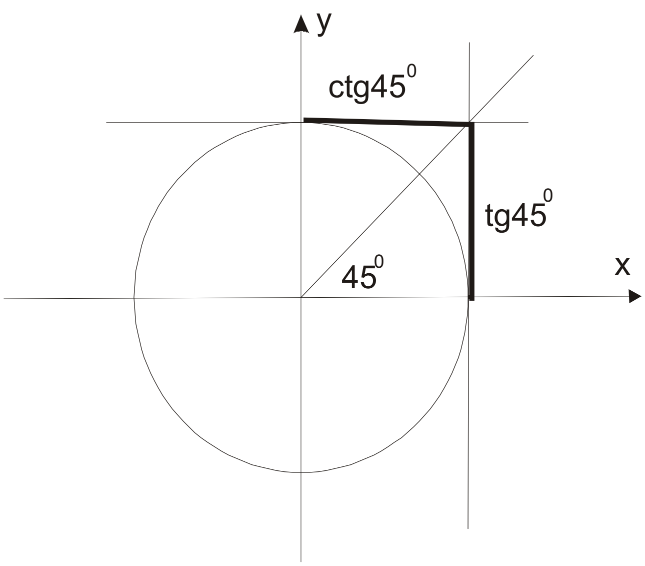


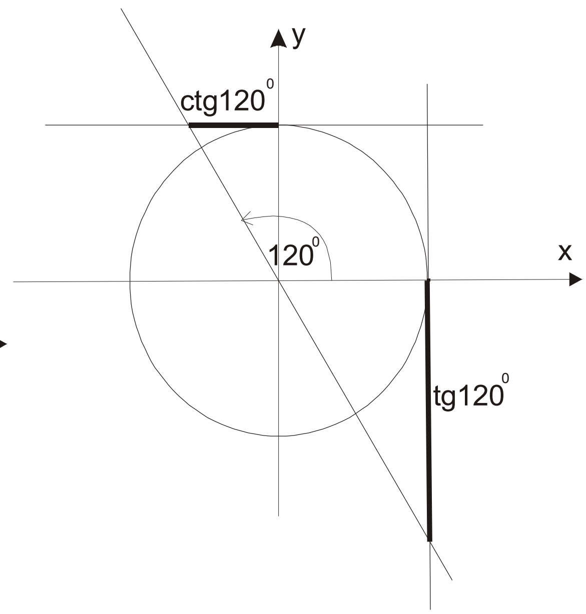
You think so as sinus and cosinuse, only **need to remember the new numbers:**

$$\frac{\sqrt{3}}{3}, \qquad 1, \qquad \sqrt{3}$$

Of course, **values for the tangent and cotangent can be positive or negative.**

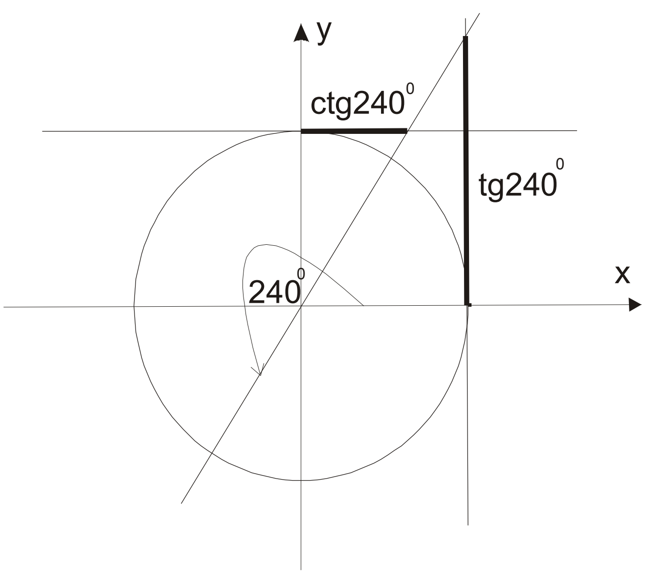
Here are a few examples:





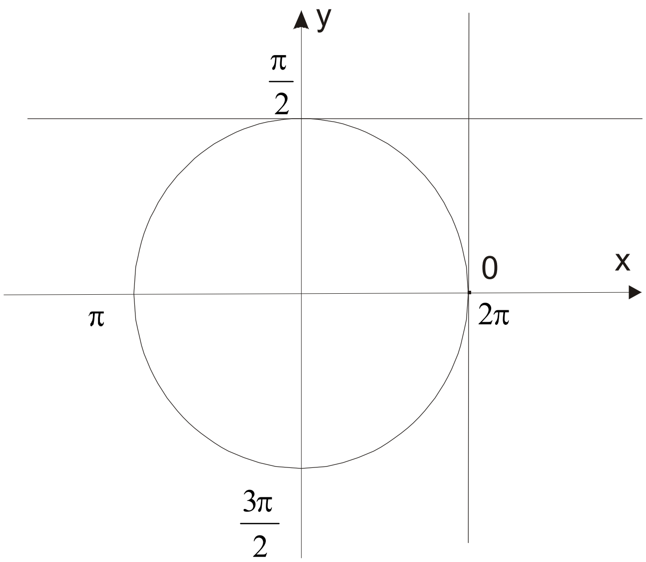
$tg45^0=1$ and $ctg45^0=1$

$tg\ 120^0=-\sqrt{3}$ and $ctg\ 120^0=-\frac{\sqrt{3}}{3}$



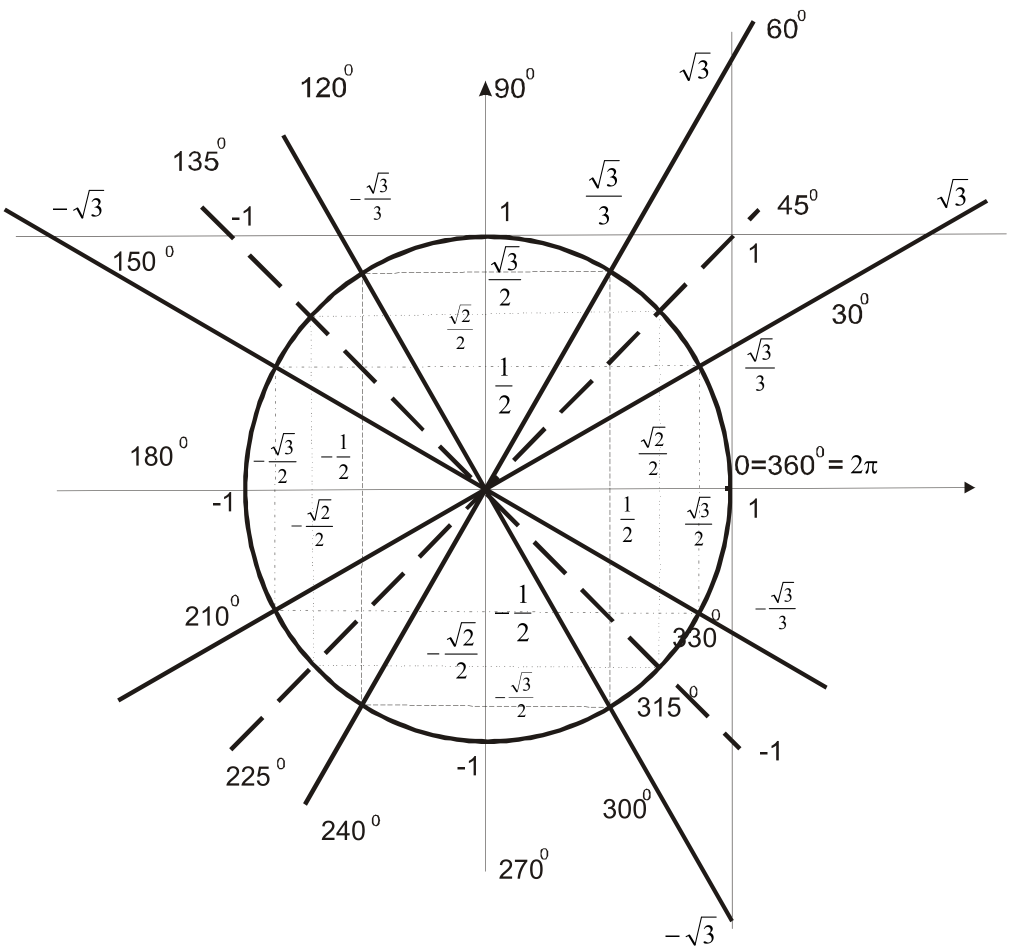
$tg240^0=\sqrt{3}$ and $ctg\ 240^0=\frac{\sqrt{3}}{3}$

What is with the boundary angles?



tg 0^0=0,   and   ctg0^0 tends to ∞ ctg90^0=0   and   tg90^0 tends to $+\infty$.

tg 180^0=0   and ctg180^0 tends to $-\infty$ ctg270^0=0   and   tg270^0 tends to $-\infty$.



Here's a small help for those who have learned to “read” on circle.